Question Number	Scheme	Notes	Marks
2(a)	$(x+2)\frac{\mathrm{d}y}{\mathrm{d}x}+y=3x^2+6x \Rightarrow$ $\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{y}{x+2}=\frac{3x(x+2)}{x+2}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}+\frac{y}{x+2}=3x$	M1: Reaches $\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{f}(x)y=\mathrm{g}(x)$ A1: Correct equation in required form. Correct answer implies both marks.	**M1** **A1**
			(2)
(b)	$\mathrm{IF}=\left\{\mathrm{e}^{\int \frac{1}{x+2}(\mathrm{d}x)}\right\}=\mathrm{e}^{\ln(x+2)}=x+2$	Correct attempt at the IF for their f(x) – must attempt the integration but need not be fully simplified (may be implied by working)	**M1**
	$"(x+2)"y=\int "3"x"(x+2)"(\mathrm{d}x)$	Applies their IF correctly with their kx	**M1**
	$(x+2)y=x^3+3x^2(+c)$	Correct equation after integration condoning omission of the constant	**A1**
	$x=4,\ y=18 \Rightarrow c=108-64-48=-4$	Substitutes correct values to find a value for c in their integrated equation	**M1**
	$y=\frac{x^3+3x^2-4}{x+2}=\frac{(x+2)(x+2)(x-1)}{x+2}$ $\Rightarrow y=(x+2)(x-1)$ or x^2+x-2	A1: Correct particular solution with g(x) in any form. Condone missing "$y=\ldots$" A1: y or $\mathrm{g}(x)=(x+2)(x-1)$ or x^2+x-2	**A1** **A1**
			(6)
			Total 8

Question Number	Scheme	Notes	Marks
6.	$\cos x\frac{\mathrm{d}y}{\mathrm{d}x}+y\sin x=(\cos^2 x)\ln x$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}+y\frac{\sin x}{\cos x}=\cos x\ln x$	Attempt to divide through by $\cos x$. If the intention is not clear, must see at least 2 terms divided by $\cos x$.	M1
	$I=\mathrm{e}^{\int\frac{\sin x}{\cos x}\mathrm{d}x}=\mathrm{e}^{-\ln\cos x}$	M1: $\mathrm{e}^{\int \pm \text{their } P(x)\,(\mathrm{d}x)}$. Dependent on the first method mark. A1: $\mathrm{e}^{-\ln\cos x}$ or $\mathrm{e}^{\ln\sec x}$	dM1A1
	$=\frac{1}{\cos x}$	$\frac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$	A1
	$\frac{y}{\cos x}=\int\ln x\,\mathrm{d}x$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{\cos x}\right)=\ln x$	M1: $y\times$ their $I=\int Q(x)\times$ their I $\mathrm{d}x$ or $\frac{\mathrm{d}}{\mathrm{d}x}(y\times\text{their } I)=Q(x)\times$ their I A1: $\frac{y}{\cos x}=\int\ln x\,\mathrm{d}x$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{\cos x}\right)=\ln x$	M1A1
	$\frac{y}{\cos x}=x\ln x-x+C$	Attempts $\int\ln x\,\mathrm{d}x$ by parts correctly (correct sign needed unless correct formula quoted and used).	M1
	$y=(x\ln x-x+C)\cos x$	Any equivalent with the constant correctly placed and "$y=\ldots$" must appear at some stage.	A1
			Total 8
	Note: Failure to divide by $\cos x$ at the start would mean that only the 3rd Method mark is available.		

4(a)	E.g. $2y\frac{dy}{dx}=-\frac{1}{t^2}\frac{dt}{dx}$ or $y^2=\frac{1}{t}\Rightarrow t=y^{-2}\Rightarrow\frac{dt}{dx}=-2y^{-3}\frac{dy}{dx}$ or $y=t^{-\frac{1}{2}}\Rightarrow\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=-\frac{1}{2}t^{-\frac{3}{2}}\frac{dt}{dx}$	B1
	$\frac{dy}{dx}+y=xy^3\Rightarrow-\frac{t^{-\frac{3}{2}}}{2}\frac{dt}{dx}+\frac{1}{\sqrt{t}}=xt^{-\frac{3}{2}}$	M1
	$\frac{dt}{dx}-2t=-2x$ *	A1*
		(3)

(b)	$I=e^{-\int 2\,dx}=e^{-2x}$	B1
	$te^{-2x}=-2\int xe^{-2x}\,dx$	M1
	$=xe^{-2x}-\int e^{-2x}\,dx$	M1
	$=xe^{-2x}+\frac{1}{2}e^{-2x}\,(+c)$	A1
	$t=\frac{1}{y^2}=x+\frac{1}{2}+ce^{2x}$ $\Rightarrow y^2=\frac{1}{x+\frac{1}{2}+ce^{2x}}\left(\text{or e.g. }\frac{2}{2x+1+Ae^{2x}}\right)$ (oe)	M1A1
		(6)
		Total 9

Alt (b)	$m-2=0\Rightarrow m=2$ so CF is $t=Ae^{2x}$	B1
	For PI try $t=ax+b$	M1
	$\frac{dt}{dx}-2t=-2x\Rightarrow a-2(ax+b)=-2x\Rightarrow a=..,b=...$	M1
	$a=1,b=\frac{1}{2}\Rightarrow t=Ae^{2x}+x+\frac{1}{2}$	A1
	$t=\frac{1}{y^2}=x+\frac{1}{2}+Ae^{2x}$ $\Rightarrow y^2=\frac{1}{x+\frac{1}{2}+Ae^{2x}}\left(\text{or e.g. }\frac{2}{2x+1+Be^{2x}}\right)$ (oe)	M1A1
		(6)

Question Number	Scheme	Notes	Marks
3(a)	$x^2\dfrac{dy}{dx}+xy=2y^2 \qquad y=\dfrac{1}{z}$		
	$\dfrac{dy}{dx}=-\dfrac{1}{z^2}\dfrac{dz}{dx}$	Correct differentiation	B1
	$-\dfrac{x^2}{z^2}\dfrac{dz}{dx}+\dfrac{x}{z}=\dfrac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\dfrac{dz}{dx}-\dfrac{z}{x}=-\dfrac{2}{x^2}$ *	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
			(3)
(a) Way 2	$y=\dfrac{1}{z}\Rightarrow zy=1\Rightarrow y\dfrac{dz}{dx}+z\dfrac{dy}{dx}=0$	Correct differentiation	B1
	$-\dfrac{y}{z}x^2\dfrac{dz}{dx}+\dfrac{x}{z}=-\dfrac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\dfrac{dz}{dx}-\dfrac{z}{x}=-\dfrac{2}{x^2}$ *	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(a) Way 3	$y=\dfrac{1}{z}\Rightarrow z=\dfrac{1}{y}\Rightarrow\dfrac{dz}{dx}=-\dfrac{1}{y^2}\dfrac{dy}{dx}$	Correct differentiation	B1
	$-\dfrac{1}{y^2}\dfrac{dy}{dx}-\dfrac{1}{xy}=-\dfrac{2}{x^2}$	Substitutes into differential equation (II)	M1
	$x^2\dfrac{dy}{dx}+xy=2y^2$	Obtains differential equation (I) with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(b)	$I=e^{-\int\frac{1}{x}dx}=e^{-\ln x}=\dfrac{1}{x}$	Correct integrating factor of $\dfrac{1}{x}$	B1
	$\dfrac{z}{x}=-\displaystyle\int\dfrac{2}{x^3}dx$	For $Iz=-\displaystyle\int\dfrac{2I}{x^2}dx$. Condone the "dx" missing.	M1
	$\dfrac{z}{x}=\dfrac{1}{x^2}+c$	Correct equation including constant	A1
	$z=\dfrac{1}{x}+cx$	Correct equation in the required form	A1
			(4)
(c)	$\dfrac{1}{y}=\dfrac{1}{x}+cx\Rightarrow-\dfrac{8}{3}=\dfrac{1}{3}+3c\Rightarrow c=-1$	Reverses the substitution and uses the given conditions to find their constant	M1

	$\dfrac{1}{y}=\dfrac{1}{x}-x\Rightarrow y=\dfrac{x}{1-x^2}$	Correct equation for y in terms of x. Allow any correct equivalents e.g. $y=\dfrac{1}{x^{-1}-x}$, $y=\dfrac{1}{\frac{1}{x}-x}$	A1
			(2)
			Total 9

4	$\dfrac{dy}{dx} - 3y\tan x = e^{4x}\sec^3 x$	
(a)	$e^{-3\int \tan x\,dx} = e^{-3\ln\sec x} = \sec^{-3} x$ or $\cos^3 x$	M1A1
	$\cos^3 x\dfrac{dy}{dx} - 3y\sin x\cos^2 x = e^{4x}\cos^3 x\sec^3 x$	
	$\dfrac{d}{dx}\left(y\cos^3 x\right) = e^{4x} \Rightarrow y\cos^3 x = \int e^{4x}\,dx$	M1
	$y\cos^3 x = \dfrac{1}{4}e^{4x} \quad (+c)$	M1
	$y = \left(\dfrac{1}{4}e^{4x} + c\right)\sec^3 x$ or $y = \left(\dfrac{1}{4}e^{4x} + c\right)\cos^{-3} x$ oe	A1
		(5)
(b)	$y = 4, \quad x = 0 \quad 4 = \left(\dfrac{1}{4} + c\right)$	
	$c = \dfrac{15}{4}$	M1
	$y = \dfrac{1}{4}\left(e^{4x} + 15\right)\sec^3 x$ or $\dfrac{1}{4}\left(e^{4x} + 15\right)\cos^{-3} x$ oe	A1
		(2)
		[7]

Question Number	Scheme	Notes	Marks
3	$\frac{dy}{dx}+2xy=xe^{-x^2}y^3$		
(a)	$z=y^{-2}\Rightarrow y=z^{-\frac{1}{2}}$		
	$\frac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1: $\frac{dy}{dx}=kz^{-\frac{3}{2}}\frac{dz}{dx}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}+\frac{2x}{z^{\frac{1}{2}}}=xe^{-x^2}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1cso
			(4)
	(a) Alternative 1		
	$\frac{dz}{dy}=-2y^{-3}$ oe	M1: $\frac{dz}{dy}=ky^{-3}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
	(a) Alternative 2		
	$\frac{dz}{dx}=-2y^{-3}\frac{dy}{dx}$	M1: $\frac{dz}{dx}=ky^{-3}\frac{dy}{dx}$ inc chain rule A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
(b)	$I=e^{\int -4x\,dx}=e^{-2x^2}$	M1: $I=e^{\int \pm 4x\,dx}$ A1: e^{-2x^2}	M1A1
	$ze^{-2x^2}=\int -2xe^{-3x^2}\,dx$	$z\times I=\int -2xe^{-x^2}I\,dx$	dM1
	$\frac{1}{3}e^{-3x^2}(+c)$	$\int xe^{qx^2}\,dx=pe^{qx^2}(+c)$	M1
	$z=ce^{2x^2}+\frac{1}{3}e^{-x^2}$	Or equivalent	A1
			(5)
(c)	$\frac{1}{y^2}=ce^{2x^2}+\frac{1}{3}e^{-x^2}\Rightarrow y^2=\frac{1}{ce^{2x^2}+\frac{1}{3}e^{-x^2}}$	$y^2=\frac{1}{(b)}\left(=\frac{3e^{x^2}}{1+ke^{3x^2}}\right)$	B1ft
			(1)
			Total 10

Question Number	Scheme	Notes	Marks
2(a)	$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+xy-x=0$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{xy}{\left(1+x^2\right)}=\frac{x}{\left(1+x^2\right)}$	Correct form.	B1
	$I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{\frac{1}{2}\ln\left(1+x^2\right)}=\left(1+x^2\right)^{\frac{1}{2}}$	M1: $I=\mathrm{e}^{\int\frac{x}{1+x^2}dx}=\mathrm{e}^{k\ln\left(1+x^2\right)}$ where k is a constant. (Condone missing brackets around the x^2+1) A1: Correct integrating factor of $\left(1+x^2\right)^{\frac{1}{2}}$	M1A1
	$y\left(1+x^2\right)^{\frac{1}{2}}=\int\frac{x}{\left(1+x^2\right)^{\frac{1}{2}}}\mathrm{d}x$	Uses their integration factor to reach the form $yI=\int Q\mathrm{I}\,\mathrm{d}x$	M1
	$=\left(1+x^2\right)^{\frac{1}{2}}(+c)$	Correct integration (+ c not needed)	A1
	$y=1+c\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao with the constant correctly placed. (The "$y=$" must appear at some point)	A1
			(6)
(b)	$2=1+c\left(1+3^2\right)^{-\frac{1}{2}}\Rightarrow c=\ldots$	Substitutes $x=3$ and $y=2$ and attempts to find a value for c.	M1
	$(y=)1+\sqrt{10}\left(1+x^2\right)^{-\frac{1}{2}}$ oe	Cao. ("$y=$" not needed for this mark) and apply isw if necessary.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
6	$2\frac{dy}{dx}+(\cot x)y+(\tan x\sec x)y^3=0$		
(a)	$z=\frac{1}{y^2}\Rightarrow\frac{dz}{dx}=-\frac{2}{y^3}\frac{dy}{dx}$ or $y^2=\frac{1}{z}\Rightarrow 2y\frac{dy}{dx}=-\frac{1}{z^2}\frac{dz}{dx}\left\{=-y^4\frac{dz}{dx}\right\}$ or $y=z^{-\frac{1}{2}}\Rightarrow\frac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	Any correct equation in $\frac{dy}{dx}$ and $\frac{dz}{dx}$ (may be implied)	B1
	$-y^3\frac{dz}{dx}+(\cot x)y+(\tan x\sec x)y^3=0$	Substitutes to obtain an equation with $\frac{dz}{dx}$ as the only derivative	M1
	$-\frac{dz}{dx}+\frac{\cot x}{y^2}+\tan x\sec x=0$ $\frac{dz}{dx}-(\cot x)z=\tan x\sec x$ *	Fully correct proof with intermediate step after substitution	A1*
			(3)
(b)	$\text{I.F.}=e^{\int-\cot x\,dx}$	$\text{I.F.}=e^{\int\pm\cot x\,dx}$	M1
	$=e^{-\ln\sin x}=\frac{1}{\sin x}$	Correct processed I.F. in any form	A1
	$\frac{z}{\sin x}=\int\frac{1}{\sin x}\times\frac{\sin x}{\cos x}\times\frac{1}{\cos x}dx$	Applies $\text{I.F.}\times z=\int\text{I.F.}\times\tan x\sec x\,dx$ Condone y for z	M1
	$\frac{z}{\sin x}=\int\sec^2 x\,dx\Rightarrow\frac{z}{\sin x}=\tan x\ \ (+c)$	Correct equation in z and x Condone missing "$+c$"	A1
	$z=\sin x(\tan x+c)\Rightarrow y^2=\frac{1}{\sin x(\tan x+c)}$ oe	Fully correct general solution. Accept equivalent forms.	A1
			(5)
(c)	$\frac{4\sqrt{3}}{3}=\frac{1}{\frac{1}{2}\left(\frac{\sqrt{3}}{3}+c\right)}\Rightarrow\frac{\sqrt{3}}{3}+c=\frac{\sqrt{3}}{2}\Rightarrow c=\ldots\left(\frac{\sqrt{3}}{6}\right)$	Correctly substitutes the given values to find a value for c	M1
	$x=\frac{\pi}{3}\Rightarrow y^2=\frac{1}{\frac{\sqrt{3}}{2}\left(\sqrt{3}+\frac{\sqrt{3}}{6}\right)}=\frac{4}{7}$ $\Rightarrow y=\pm\frac{2\sqrt{7}}{7}$ oe	**dM1**: Substitutes $x=\frac{\pi}{3}$ and their c into their equation and reaches $y^2=\ldots$ or $y=$ with trig terms evaluated. A1: Both correct exact values. The negative value must not be rejected.	**dM1 A1**
			(3)

Question Number	Scheme	Notes	Marks
8(a)	Allow "single fraction" to be implied by sum/difference of fractions with same denominator or a product of fractions. No further fractions in numerator/denominator.		

	$\cot 2x\{+\tan x\} = \frac{\cos 2x}{\sin 2x}\left\{+\frac{\sin x}{\cos x}\right\}$	Uses $\cot 2x = \frac{\cos 2x}{\sin 2x}$ or e.g., $\frac{\cos 2x}{2\sin x\cos x}$	**M1**
	$\frac{\cos 2x + 2\sin^2 x}{2\sin x\cos x} \Rightarrow$ e.g., $\frac{1-2\sin^2 x+2\sin^2 x}{2\sin x\cos x}$ or $\frac{\cos^2 x-\sin^2 x+2\sin^2 x}{2\sin x\cos x}$ $\frac{2\cos^2 x-1+2\sin^2 x}{\sin 2x}$ or $\frac{\cos 2x+1-\cos 2x}{\sin 2x}$ OR $\frac{\cos 2x+\tan x\sin 2x}{\sin 2x} \Rightarrow$ $\Rightarrow \frac{\cos 2x+\frac{\sin x}{\cos x}\times 2\sin x\cos x}{\sin 2x} \Rightarrow$ e.g., $\frac{1-2\sin^2 x+2\sin^2 x}{\sin 2x}$ OR $\frac{\cos 2x\cos x+\sin x\sin 2x}{\sin 2x\cos x} \Rightarrow$ $\frac{\cos x}{\sin 2x\cos x}$ or $\frac{\cos^3 x-\sin^2 x\cos x+2\sin^2 x\cos x}{\sin 2x\cos x}$	Uses sufficient correct identities e.g., $\cos 2x = 1-2\sin^2 x$ $\cos 2x = \cos^2 x-\sin^2 x$ $\cos 2x = 2\cos^2 x-1$ $2\sin^2 x = 1-\cos 2x$ $\cos 2x\cos x+\sin x\sin 2x = \cos(2x-x)$ to obtain a correct single fraction with numerator in terms of sin x and/or cos x or "$\cos 2x+1-\cos 2x$". A qualifying fraction must be seen before $\frac{1}{2\sin x\cos x}$ or $\frac{1}{\sin 2x}$ Condone poor notation.	**A1** **(M1 on ePen)**
	$= \frac{1}{2\sin x\cos x}$ or $\frac{1}{\sin 2x} = \operatorname{cosec} 2x$ *	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. $\sin x^2$ for this mark.	**A1***
			(3)
Alt	$\cot 2x\{+\tan x\} = \frac{1-\tan^2 x}{2\tan x}\{+\tan x\}$	Uses $\cot 2x = \frac{1-\tan^2 x}{2\tan x}$	**M1**
	$\frac{1-\tan^2 x+2\tan^2 x}{2\tan x} \Rightarrow$ e.g., $\frac{\tan^2 x+1}{2\tan x} \Rightarrow \frac{\left(\frac{\sin x}{\cos x}\right)^2+1}{2\frac{\sin x}{\cos x}} \Rightarrow \frac{\cos x(\sin^2 x+\cos^2 x)}{2\cos^2 x\sin x}$ or $\frac{\tan^2 x+1}{2\tan x}\left\{\times\frac{\cos x}{\cos x}\right\} \Rightarrow \frac{\sin^2 x+\cos^2 x}{2\sin x\cos x}$ or $\frac{\sec^2 x}{2\tan x}$ or $\frac{\cos x}{2\cos^2 x\sin x}$	Uses correct identities e.g., $\tan x = \frac{\sin x}{\cos x}$ oe to obtain a correct single fraction in sin x and cos x but allow $\frac{\sec^2 x}{2\tan x}$ following use of $\sec^2 x = 1+\tan^2 x$ A qualifying fraction must be seen before $\frac{1}{2\sin x\cos x}$ or $\frac{1}{\sin 2x}$ Condone poor notation.	**A1** **(M1 on ePen)**
	$\frac{1}{2\sin x\cos x}$ or $\frac{1}{\sin 2x} = \operatorname{cosec} 2x$ *	Fully correct proof with one of the two intermediate fractions seen. All notation correct – no mixed or missing arguments or e.g. $\sin x^2$ for this mark.	**A1***
			(3)

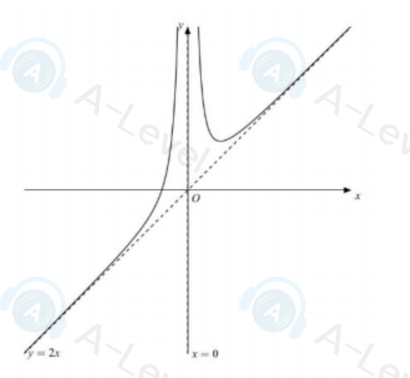
Question Number	Scheme	Notes	Marks
8(b)	Examples:		**M1** **A1**

	$y^2 = w\sin 2x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}w}{\mathrm{d}x}\sin 2x + 2w\cos 2x$ or $y = w^{\frac{1}{2}}(\sin 2x)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}w^{\frac{1}{2}}(\sin 2x)^{-\frac{1}{2}}(2\cos 2x) + \frac{1}{2}w^{-\frac{1}{2}}\frac{\mathrm{d}w}{\mathrm{d}x}(\sin 2x)^{\frac{1}{2}}$ or $w = \frac{y^2}{\sin 2x} \Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} = \frac{2y\sin 2x\frac{\mathrm{d}y}{\mathrm{d}x} - y^2.2\cos 2x}{\sin^2 2x}$ or $w = y^2\mathrm{cosec}2x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{cosec}2x - 2y^2\mathrm{cosec}2x\cot 2x$ **M1**: Attempts the differentiation of the given substitution using the product/quotient and chain rules **and** obtains an **equation** in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}w}{\mathrm{d}x}$ of the correct form (sign/coefficient errors only and allow sign errors with quotient/product rule). This mark is not available for work in $\frac{\mathrm{d}y}{\mathrm{d}w}$ or $\frac{\mathrm{d}w}{\mathrm{d}y}$ unless appropriate work follows to achieve an equation in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}w}{\mathrm{d}x}$ of the correct form. **A1**: Correct differentiation	
	$y\frac{\mathrm{d}y}{\mathrm{d}x} + y^2\tan x = \sin x \rightarrow$ e.g., $\frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\sin 2x + 2w\cos 2x\right) + w\sin 2x\tan x = \sin x$ A recognisable attempt to eliminate y from the original equation to obtain an equation involving $\frac{\mathrm{d}w}{\mathrm{d}x}$, w and x only . Not dependent.	**M1**
	$\Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} + 2w(\cot 2x + \tan x) = \frac{2\sin x}{\sin 2x}$ $\Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} + 2w\mathrm{cosec}\ 2x = \sec x$ * Fully correct work leading to the given equation with $2w(\cot 2x + \tan x)$ or e.g., $2w\cot 2x + 2w\tan x$ clearly replaced by $2w$ cosec $2x$ but allow cot $2x$ written as $\frac{1}{\tan 2x}$ or $\frac{\cos 2x}{\sin 2x}$ and/or tan x written as $\frac{\sin x}{\cos x}$ If the result in (a) is not clearly used there must be full equivalent work. Allow use of "csc $2x$"	**A1***
		(4)

8(c)	$\frac{\mathrm{d}w}{\mathrm{d}x} + 2w\,\mathrm{cosec}\,2x = \sec x \Rightarrow \mathrm{IF} = \mathrm{e}^{2\int \mathrm{cosec}\,2x\,\mathrm{d}x} = \tan x$ or $\mathrm{e}^{-\ln(\mathrm{cosec}2x + \cot 2x)} \Rightarrow \frac{1}{\mathrm{cosec}2x + \cot 2x}$ or $\frac{1}{\cot x}$ or $\tan x$	**M1**: $\mathrm{e}^{2\int \mathrm{cosec}\,2x(\mathrm{d}x)}$ condoning omission of one or both "2"s **A1**: tan x oe Allow $k\tan x$ e.g., $\mathrm{e}^{2c}\tan x$ Not just $\mathrm{e}^{\ln(\tan x)}$	**M1** **A1**
	$\Rightarrow w\text{"}\tan x\text{"} = \int \text{"}\tan x\text{"}\sec x\ \{\mathrm{d}x\}$	Correctly applies their integrating factor to the equation, i.e., $\Rightarrow \mathrm{IF}\times w = \int \mathrm{IF}\times\sec x\ \{\mathrm{d}x\}$ Allow equivalents for sec x. Condone "y" used for "w" for this mark.	**M1**
	$\Rightarrow w\tan x = \sec x(+c)$	Correct equation oe with or without constant.	**A1**

	Using IF $= \dfrac{1}{\operatorname{cosec}2x+\cot 2x} \Rightarrow$ RHS of $\displaystyle\int \frac{\sec x}{\operatorname{cosec}2x+\cot 2x}\,dx$ which is likely to need rewriting as $\int \tan x \sec x\,dx$ Note that IBP on $\sec x \tan x$ by writing it as $\sec^2 x \sin x$ can lead to $\sin x \tan x + \cos x (+c)$ Use Review for any attempts at integration you are unsure about.	
	e.g., $y^2 = w\sin 2x$ and $w\tan x = \sec x + c \Rightarrow \dfrac{y^2}{\sin 2x}\tan x = \sec x + c$ $\Rightarrow y^2 = ... \quad \left\{\dfrac{\sin 2x}{\tan x}(\sec x + c)\right\}$ Substitutes for w correctly and reaches $y^2 = \ldots$ Their $y^2 = \ldots$ must be consistent with **their** equation in w and x that immediately followed their integration. **This mark requires both previous M marks and an attempt at integration that includes a "+ c"** A further example is: $w = \operatorname{cosec} x + \dfrac{c}{\tan x} \Rightarrow y^2 = \operatorname{cosec} x \sin 2x + \dfrac{c\sin 2x}{\tan x}$	**ddM1**
	$\left\{\text{e.g., } y^2 = \dfrac{2\sin x\cos^2 x}{\sin x}\left(\dfrac{1}{\cos x}+c\right) \Rightarrow\right\}$ $y^2 = 2\cos x + A\cos^2 x$ Any correct $y^2 = \ldots$ equation with RHS fully in terms of $\cos x$. E.g. accept $y^2 = 2\cos x + 2c\cos^2 x \quad y^2 = \cos x(2 + A\cos x) \quad y^2 = 2\cos^2 x\left(\dfrac{1}{\cos x}+c\right)$ Ignore any inconsistencies with the constant e.g., $2c$ later written as c	**A1**
		(6)
		Total 13

4 (a)	$y^2 = z^{-1} \Rightarrow 2y\dfrac{dy}{dx} = -\dfrac{1}{z^2}\dfrac{dz}{dx}$ oe eg $\dfrac{dy}{dx} = -\dfrac{1}{2}z^{-\frac{3}{2}}\dfrac{dz}{dx}$	B1	
	$2y\dfrac{dy}{dx} + 4y^2 = 6xy^4$		
	$-\dfrac{1}{z^2}\dfrac{dz}{dx} + \dfrac{4}{z} = \dfrac{6x}{z^2}$	M1	
	$\dfrac{dz}{dx} - 4z = -6x$ *	A1 *	(3)
(b)	$\text{IF} = e^{\int -4dx} = e^{-4x}$	B1	
	$e^{-4x}\left(\dfrac{dz}{dx} - 4z\right) = e^{-4x} \times -6x$		
	$ze^{-4x} = -6\int xe^{-4x}dx$	M1	
	$= -6\left[-\dfrac{1}{4}xe^{-4x} + \int \dfrac{1}{4}e^{-4x}dx\right]$	M1	
	$= -6\left[-\dfrac{1}{4}xe^{-4x} - \dfrac{1}{16}e^{-4x}\right](+c)$ oe	A1	
	$= \dfrac{3}{2}xe^{-4x} + \dfrac{3}{8}e^{-4x}(+c)$		
	$z = \dfrac{3}{2}x + \dfrac{3}{8} + ce^{4x}$ oe	A1	(5)
ALT	$\dfrac{dz}{dx} - 4z = -6x$		
	$m - 4 = 0 \Rightarrow m = 4 \Rightarrow$ CF is $z = Ae^{4x}$	B1	
	PI: $z = \lambda + \mu x$	M1	
	$\dfrac{dz}{dx} = \mu \Rightarrow \mu - 4(\lambda + \mu x) = -6x$		
	$4\mu = 6 \quad 4\lambda = \mu, \Rightarrow \mu = \dfrac{3}{2}, \lambda = \dfrac{3}{8}$	M1,A1	
	$z = \dfrac{3}{2}x + \dfrac{3}{8} + Ae^{4x}$	A1	
(c)	$y^2 = \dfrac{1}{\dfrac{3}{2}x + \dfrac{3}{8} + ce^{4x}} = \dfrac{8}{(12x + 3 + Ae^{4x})}$ oe	B1ft	(1)
			[9]

8(a)	$\frac{dv}{dx}=\frac{dy}{dx}-2$	**B1**
	$\frac{dy}{dx}+2yx(y-4x)=2-8x^3 \to \frac{dv}{dx}+2+2(v+2x)x(v+2x-4x)=2-8x^3$	
	$\to \frac{dv}{dx}+2+2x\left(v^2-4x^2\right)=2-8x^3$	**M1**
	$\to \frac{dv}{dx}=-2xv^2$ *	**A1***
		(4)
(b)	$\frac{1}{v^2}\frac{dv}{dx}=-2x \Rightarrow \int v^{-2}\,dv=-2\int x\,dx$	**B1**
	$\Rightarrow \frac{v^{-1}}{-1}=-2\frac{x^2}{2}(+c)$	**M1**
	$\Rightarrow \frac{1}{v}=x^2+c$	**A1**
	$\Rightarrow v=\frac{1}{x^2+c}$	**A1**
		(4)
(c)	$y=2x+\frac{1}{x^2+c}$	**B1ft**
		(1)
(d)	$-1=2\times-1+\frac{1}{1+c}\Rightarrow c=\ldots$	**M1**
	$y=2x+\frac{1}{x^2}$	**A1**
	 Attempts the sketch for their equation, with at least one of - One branch correct - Vertical asymptote for their equation - Long term behaviour tends to infinity - Minimum in quadrant 1	**M1**
	Fully correct shape, two branches tending to infinity as x tends to infinity both directions, with minimum in first quadrant No need for oblique asymptote marked.	**A1**
	y-axis a vertical asymptote labelled	**B1ft**
		(5)
		(14 marks)