Question Number	Scheme		Marks
5	$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$		
(a)	$y'''-2y'-2xy''+2y'(=0)\left(y'''=2xy''\right)$	M1: Attempt to differentiate including use of the product rule on $2x\frac{dy}{dx}$ Equation may have been re-written as $y''=$ ... before differentiating	M1A1
		A1: Correct differentiation	
	$y''''-2y''-2xy'''-2y''+2y''(=0)$	M1: Second use of product rule. Dependent on first M1.	dM1A1
		A1: Correct differentiation NB A simpler form is obtained if $y'''-2xy''=0$ is used.	
	$y''''=2xy'''+2y''$ $=2x\left(2xy''\right)+2y''=\left(4x^2+2\right)y''$	Cao and cso	A1
			(5)
(b)	$y_0''=-2,\ \ y_0'''=0\ \ \ y_0^{iv}=-4$	B1: $y_0''=-2$	B1, M1A1
		M1: Attempts y_0''' and y_0^{iv}	
		A1: All correct and obtained from correct expressions	
	$(y=)1+3x-x^2-\frac{x^4}{6}$	M1: Correct use of Maclaurin series	M1A1
		A1: Fully correct expansion.	
			(5)
(c)	$(y=)1+3(-0.2)-(-0.2)^2-\frac{(-0.2)^4}{6}$	Use of the correct Maclaurin series and substitution of $x=-0.2$	M1
	$(y=)\ 0.3597$	Allow awrt	A1
			(2)
			Total 12

7(a)	$y=e^x\sin x\Rightarrow\left(\frac{dy}{dx}=\right)e^x\sin x+e^x\cos x$ $\Rightarrow\left(\frac{d^2y}{dx^2}=\right)e^x\sin x+e^x\cos x+e^x\cos x-e^x\sin x=2e^x\cos x$ (oe – see notes)	M1
	E.g. (see notes for equivalents) $\Rightarrow\left(\frac{d^3y}{dx^3}=\right)2e^x\cos x-2e^x\sin x$ $\Rightarrow\frac{d^4y}{dx^4}=2e^x\cos x-2e^x\sin x-2e^x\cos x-2e^x\sin x$	dM1
	$\frac{d^4y}{dx^4}=-4e^x\sin x=-4y$ (oe – see notes)	A1
	$\Rightarrow\frac{d^4y}{dx^4}=-4y\Rightarrow\frac{d^5y}{dx^5}=-4\frac{dy}{dx}\Rightarrow\frac{d^6y}{dx^6}=-4\frac{d^2y}{dx^2}$	A1
		(4)

(b)	$(y)_0=0,\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0=1,\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0=2,\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0=2,\left(\frac{\mathrm{d}^4y}{\mathrm{d}x^4}\right)_0=0,\left(\frac{\mathrm{d}^5y}{\mathrm{d}x^5}\right)_0=-4,\left(\frac{\mathrm{d}^6y}{\mathrm{d}x^6}\right)_0=-8$	M1
	$(y=)0+x+\frac{x^2}{2}\times2+\frac{x^3}{3!}\times2+0-\frac{x^5}{5!}\times4-\frac{x^6}{6!}\times8+\ldots$	M1
	$(y=)x+x^2+\frac{x^3}{3}-\frac{x^5}{30}-\frac{x^6}{90}+\ldots$	A1
		(3)
		Total 7

4 (a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3$	M1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{4}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\frac{8}{y}\times\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{4}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3+\frac{8}{y}\left(\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{28}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\frac{24}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)
ALT	$\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-8\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1A1A1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{y}\left(7\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{4}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3\right)-\frac{3}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{28}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\frac{24}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	A1* (5)
4(b)	At $x=0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{4}{8}(1)^2-3=-\frac{5}{2}$ oe	B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{28}{64}\times1^3-\frac{24}{8}\times1=-\frac{41}{16}$	M1
	$y=8+x-\frac{5}{2}\times\frac{x^2}{2!}-\frac{41}{16}\times\frac{x^3}{3!}+\ldots$	M1
	$y=8+x-\frac{5}{4}x^2-\frac{41}{96}x^3+\ldots$	A1 (4) [9]

5	$y=\mathrm{e}^{\cos^2 x}$		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin x\cos x\mathrm{e}^{\cos^2 x}=-\sin 2x\mathrm{e}^{\cos^2 x}$ M1: Differentiates using the chain rule to obtain an expression of the form $\alpha\sin x\cos x\mathrm{e}^{\cos^2 x}$ or $\beta\sin 2x\mathrm{e}^{\cos^2 x}$ A1: Correct derivative Note that candidates may use $\frac{1}{2}(1+\cos 2x)$ instead of $\cos^2 x$		M1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\sin 2x(-2\sin x\cos x\mathrm{e}^{\cos^2 x})-2\cos 2x\mathrm{e}^{\cos^2 x}$ Correct use of the Product Rule on their first derivative **Dependent on the first method mark.**		**dM1**
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{\cos^2 x}(\sin^2 2x-2\cos 2x)$ *	Achieves the printed answer with no errors.	A1*
	Alternative using logs: $y=\mathrm{e}^{\cos^2 x}\Rightarrow \ln y=\cos^2 x\Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin x\cos x$ M1: $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=k\sin x\cos x$ or $k\sin 2x$ A1: $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin x\cos x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-y\sin 2x\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{\mathrm{d}y}{\mathrm{d}x}\sin 2x-2y\cos 2x$ M1: Correct use of product rule $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{\cos^2 x}(\sin^2 2x-2\cos 2x)$ * A1: Achieves the printed answer with no errors.		
			(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\mathrm{e}$	Both seen, can be implied by subsequent work.	B1
	$\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2}\mathrm{f}''(0)+\ldots$ $=\mathrm{e}^{\cos^2 0}-\sin 0\mathrm{e}^{\cos^2 0}x+\frac{1}{2}\mathrm{e}^{\cos^2 0}(\sin^2 0-2\cos 0)x^2+\ldots$ Applies the **correct** Maclaurin expansion, the "$\frac{1}{2}$" is required and there must be no x's in the derivatives. This can be implied by their expansion but if the expansion in incorrect for their values and the formula is not quoted, score M0.		M1
	$\left(\mathrm{e}^{\cos^2 x}\right)=\mathrm{e}(1-x^2+\ldots)$	Or exact equivalent e.g. $\mathrm{e}-\mathrm{e}x^2$ (i.e. all trig. evaluated)	A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
4(a)	$\mathrm{f}(x)=\sin\left(\tfrac{3}{2}x\right)$ $\mathrm{f}'(x)=\tfrac{3}{2}\cos\left(\tfrac{3}{2}x\right)$ $\mathrm{f}''(x)=-\tfrac{9}{4}\sin\left(\tfrac{3}{2}x\right)$ $\mathrm{f}'''(x)=-\tfrac{27}{8}\cos\left(\tfrac{3}{2}x\right)$ $\mathrm{f}''''(x)=\tfrac{81}{16}\sin\left(\tfrac{3}{2}x\right)$	M1: Attempt first 4 derivatives. Should be $\sin\to\cos\to\sin\to\cos\to\sin$. I.e. ignore signs and coefficients. A1: $\mathrm{f}'=\tfrac{3}{2}\cos\left(\tfrac{3}{2}x\right)$ and $\mathrm{f}''=-\tfrac{9}{4}\sin\left(\tfrac{3}{2}x\right)$ A1: $\mathrm{f}'''=-\tfrac{27}{8}\cos\left(\tfrac{3}{2}x\right)$ and $\mathrm{f}''''=\tfrac{81}{16}\sin\left(\tfrac{3}{2}x\right)$ Allow un-simplified e.g. $\mathrm{f}''=-\tfrac{3}{2}.\tfrac{3}{2}\sin\left(\tfrac{3}{2}x\right)$	M1A2
	$y\left(\tfrac{\pi}{3}\right)=1, y'\left(\tfrac{\pi}{3}\right)=0, y''\left(\tfrac{\pi}{3}\right)=-\tfrac{9}{4}, y'''\left(\tfrac{\pi}{3}\right)=0, y''''\left(\tfrac{\pi}{3}\right)=\tfrac{81}{16}$ Attempts at least 1 derivative at $x=\dfrac{\pi}{3}$		M1
	$\mathrm{f}(x)=1-\dfrac{9}{8}\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{27}{128}\left(x-\dfrac{\pi}{3}\right)^4$	dM1: Correct use of Taylor series. I.e. $\mathrm{f}(x)=\mathrm{f}\left(\tfrac{\pi}{3}\right)+\left(x-\tfrac{\pi}{3}\right)\mathrm{f}'\left(\tfrac{\pi}{3}\right)+\left(x-\tfrac{\pi}{3}\right)^2\dfrac{\mathrm{f}''\left(\tfrac{\pi}{3}\right)}{2!}+\ldots$ Evidence of at least one term of the correct structure i.e. $\left(x-\tfrac{\pi}{3}\right)^n\dfrac{\mathrm{f}^n\left(\tfrac{\pi}{3}\right)}{n!}$ and not a Maclaurin series. **Dependent on the previous method mark.** A1: Correct expansion. Allow equivalent **single** fractions for $\dfrac{9}{8}$ and/or $\dfrac{27}{128}$ and allow decimal equivalents i.e. 1.125 and 0.2109375. Ignore any extra terms.	dM1A1
			(6)
(b)	$\mathrm{f}\left(\dfrac{1}{3}\right)=0.4815$	M1: Attempts $\mathrm{f}\left(\dfrac{1}{3}\right)$ or states $x=\dfrac{1}{3}$ A1: 0.4815 cao	M1A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
3	$y=\arcsin 2x \quad \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2}{\sqrt{1-4x^2}}\left\{=2\left(1-4x^2\right)^{-\frac{1}{2}}\right\}$		
(a)	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\left(1-4x^2\right)^{-\frac{3}{2}}(-8x) \quad \left\{=8x\left(1-4x^2\right)^{-\frac{3}{2}}\right\}$	Obtains correct form for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1**
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=(8x)\left(-\dfrac{3}{2}\right)\left(1-4x^2\right)^{-\frac{5}{2}}(-8x)+8\left(1-4x^2\right)^{-\frac{3}{2}}$ or $\dfrac{8\left(1-4x^2\right)^{\frac{3}{2}}+96x^2\left(1-4x^2\right)^{\frac{1}{2}}}{\left(1-4x^2\right)^3}$	Uses product/quotient rule to obtain correct form for $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$	**dM1**
	$=\dfrac{96x^2+8\left(1-4x^2\right)}{\left(1-4x^2\right)^{\frac{5}{2}}}=\dfrac{64x^2+8}{\left(1-4x^2\right)^{\frac{5}{2}}}$	Correct answer in the correct form	**A1**
			(3)
	Allow full access to the remaining marks with an incorrect value of A		
(b)	$y(0)=0,\ y'(0)=2,\ y''(0)=0,\ y'''(0)=8$ $\mathrm{f}(x)=\mathrm{f}(0)+\mathrm{f}'(0)x+\dfrac{\mathrm{f}''(0)}{2}x^2+\dfrac{\mathrm{f}'''(0)}{3!}x^3+\ldots$ $(\Rightarrow \arcsin 2x)\approx 2x+\dfrac{4}{3}x^3$	M1: Finds values for $y(0),\ y'(0),\ y''(0)$ and $y'''(0)$ and obtains a consistent Maclaurin expansion A1: Correct expression	**M1** **A1**
			(2)
(c)	$\mathrm{e}^{3x}\arcsin 2x=(1+3x+\ldots)\left(\text{"}2x+\dfrac{4}{3}x^3+\ldots\text{"}\right)=\ldots$	Attempts to find the expansion of e^{3x} and multiplies by their expression for $\arcsin 2x$	**M1**
	$=\left(1+3x+\dfrac{(3x)^2}{2}+\ldots\right)\left(\text{"}2x+\dfrac{4}{3}x^3+\ldots\text{"}\right)=\ldots$ $=2x+6x^2+9x^3+\dfrac{4}{3}x^3+\ldots$	For a **correct** expansion of e^{3x} up to the x^2 term multiplied by their expression for arcsin with at least one correct term obtained.	**A1**
	$\Rightarrow \mathrm{e}^{3x}\arcsin 2x\approx 2x+6x^2+\dfrac{31}{3}x^3$	Fully correct expression	**A1**
			(3)
			Total 8

3(a)	$2x\frac{d^2y}{dx^2}+y\frac{dy}{dx}-6xy=0$ and $\frac{dy}{dx}=3$, $y=\frac{1}{2}$ when $x=2$	
	$\frac{d^2y}{dx^2}=\frac{1}{2x}\left(6xy-y\frac{dy}{dx}\right)\Rightarrow\left.\frac{d^2y}{dx^2}\right\vert_{x=2}=\frac{1}{4}\left(6-\frac{3}{2}\right)=\frac{9}{8}$	**M1A1**
	E.g. $2\frac{d^2y}{dx^2}+2x\frac{d^3y}{dx^3}+\underline{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2}-6y-6x\frac{dy}{dx}=0$ Or $\frac{d^3y}{dx^3}=-\frac{1}{2x^2}\left(6xy-y\frac{dy}{dx}\right)+\frac{1}{2x}\left(6x\frac{dy}{dx}+6y-\underline{\left(\frac{dy}{dx}\right)^2-y\frac{d^2y}{dx^2}}\right)$ Or $2x\frac{d^2y}{dx^2}=-y\frac{dy}{dx}+6xy\Rightarrow 2\frac{d^2y}{dx^2}+2x\frac{d^3y}{dx^3}=-\underline{\left(\frac{dy}{dx}\right)^2-y\frac{d^2y}{dx^2}}+6y+6x\frac{dy}{dx}$	**M1**$\underline{\textbf{A1}}$ **A1**
	$\left.\frac{d^3y}{dx^3}\right\vert_{x=2}=\frac{435}{64}$	**A1**
		(6)
(b)	$(y)=\frac{1}{2}+3(x-2)+\frac{9/8}{2}(x-2)^2+\frac{435/64}{3!}(x-2)^3$	**M1**
	$y=\frac{1}{2}+3(x-2)+\frac{9}{16}(x-2)^2+\frac{145}{128}(x-2)^3$	**A1**
		(2)
		(8 marks)

5. **(a)**	$\frac{d^2y}{dx^2}=-\frac{2}{y}\left(\frac{dy}{dx}\right)^2+2$	
	$\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$ seen	B1
	$\frac{d^3y}{dx^3}=-\frac{4}{y}\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}+\frac{2}{y^2}\left(\frac{dy}{dx}\right)^3$	M1A1A1
		(4)
ALT:	$\left(\frac{dy}{dx}\right)^2\to 2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$ seen $\underline{\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)+y\frac{d^3y}{dx^3}}+4\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)\underline{-2\frac{dy}{dx}}=0$ $\frac{d^3y}{dx^3}=\frac{1}{y}\left(-5\frac{d^2y}{dx^2}+2\right)\frac{dy}{dx}$	B1 M1$\underline{\text{A1}}$ A1 (4)
(b)	At $x=0$ $\frac{d^2y}{dx^2}=\frac{1}{2}\left(-2\times(1)^2+4\right)=1$	B1
	$\frac{d^3y}{dx^3}=\frac{1}{2}(-5\times1+2)\times1=\frac{-3}{2}$	M1
	$(y=)2+x+(1)\frac{x^2}{2!}+\left(\frac{-3}{2}\right)\frac{x^3}{3!}+\ldots$	M1
	$y=2+x+\frac{1}{2}x^2-\frac{1}{4}x^3+\ldots$	A1 (4)
		[8]

5(a)	$y=\tan^2 x \Rightarrow \dfrac{dy}{dx}=2\tan x\sec^2 x$	Correct first derivative any correct form	B1
	$\dfrac{dy}{dx}=2\tan x\sec^2 x \Rightarrow \dfrac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x$ M1: Correct application of the product rule and chain rule A1: Correct expression		M1A1
	$\dfrac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x \Rightarrow \dfrac{d^3y}{dx^3}=8\sec^4 x\tan x+8\sec^2 x\tan^3 x+8\sec^4 x\tan x$ Or $\dfrac{d^2y}{dx^2}=6\sec^4 x-4\sec^2 x \Rightarrow \dfrac{d^3y}{dx^3}=24\sec^4 x\tan x-8\sec^2 x\tan x$ M1: Attempt to differentiate using product and chain rule. At least one term to be correct		M1
	$=8\sec^4 x\tan x+8\sec^2 x\tan x\left(\sec^2 x-1\right)+8\sec^4 x\tan x$ $=24\sec^4 x\tan x-8\sec^2 x\tan x=8\sec^2 x\tan x\left(3\sec^2 x-1\right)$ Fully correct expression		A1
			(5)
(b)	$(y)_{\frac{\pi}{3}}=3,\ (y')_{\frac{\pi}{3}}=8\sqrt{3},\ (y'')_{\frac{\pi}{3}}=80,\ (y''')_{\frac{\pi}{3}}=352\sqrt{3}$	Attempts the values up to the third derivative when $x=\dfrac{\pi}{3}$	M1
	$y=3+8\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+\dfrac{80}{2!}\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{352\sqrt{3}}{3!}\left(x-\dfrac{\pi}{3}\right)^3+\ldots$ Correct application of the Taylor series 2! or 2, 3! or 6		M1
	$y=3+8\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+40\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{176\sqrt{3}}{3}\left(x-\dfrac{\pi}{3}\right)^3+\ldots$ Correct expansion Must start $y=$ ... or $\tan^2 x=$ .... f(x) only accepted if f(x) has been defined to be $\tan^2 x$		A1
			(3)
			Total 8

4(a)	$y=\tan\left(\dfrac{3x}{2}\right)\Rightarrow y'=\dfrac{3}{2}\sec^2\left(\dfrac{3x}{2}\right)$	Any correct first derivative. Not implied by $y'\left(\frac{\pi}{6}\right)=3$	**B1**
	$\Rightarrow y''=2\times\dfrac{3}{2}\sec\left(\dfrac{3x}{2}\right)\times\sec\left(\dfrac{3x}{2}\right)\tan\left(\dfrac{3x}{2}\right)\times\dfrac{3}{2}$ $\left[=\dfrac{9}{2}\sec^2\left(\dfrac{3x}{2}\right)\tan\left(\dfrac{3x}{2}\right)\right]$	Attempts the second derivative achieving $k\sec^2\left(\dfrac{3x}{2}\right)\tan\left(\dfrac{3x}{2}\right)$ or unsimplified equivalent. Not implied by $y''\left(\frac{\pi}{6}\right)=9$	**M1**
	$\Rightarrow y'''=\dfrac{9}{2}\sec^2\left(\dfrac{3x}{2}\right)\sec^2\left(\dfrac{3x}{2}\right)\times\dfrac{3}{2}+\dfrac{9}{2}\tan\left(\dfrac{3x}{2}\right)\times 2\times\dfrac{3}{2}\sec^2\left(\dfrac{3x}{2}\right)\tan\left(\dfrac{3x}{2}\right)$ $\left[=\dfrac{27}{4}\sec^4\left(\dfrac{3x}{2}\right)+\dfrac{27}{2}\sec^2\left(\dfrac{3x}{2}\right)\tan^2\left(\dfrac{3x}{2}\right)\right]$	**d**M1: Attempts third derivative using the product rule, achieving $P\sec^4\left(\dfrac{3x}{2}\right)+Q\sec^2\left(\dfrac{3x}{2}\right)\tan^2\left(\dfrac{3x}{2}\right)$ or unsimplified equivalent. **Requires previous M mark.** A1: Correct differentiation. Accept unsimplified. Not implied by $y'''\left(\frac{\pi}{6}\right)=54$	**dM1** **A1**
	If $\sec^2\left(\dfrac{3x}{2}\right)=\tan^2\left(\dfrac{3x}{2}\right)+1$ is used the identity must be used correctly and to score M marks expressions of consistent form should be achieved. Note that replacing $\sec^2\left(\dfrac{3x}{2}\right)$ in $y''\Rightarrow y'''=\dfrac{27}{4}\sec^2\left(\dfrac{3x}{2}\right)+\dfrac{81}{4}\sec^2\left(\dfrac{3x}{2}\right)\tan^2\left(\dfrac{3x}{2}\right)$		

	$y\left(\frac{\pi}{6}\right)=1, y'\left(\frac{\pi}{6}\right)=3, y''\left(\frac{\pi}{6}\right)=9, y'''\left(\frac{\pi}{6}\right)=54$ Attempts values (but allow numerical trig expressions) for y and their 3 derivatives at $\frac{\pi}{6}$ - accept stated values or insertion into a series of the correct form		M1
	$(y=)1+3\left(x-\frac{\pi}{6}\right)+\frac{9}{2!}\left(x-\frac{\pi}{6}\right)^2+\frac{54}{3!}\left(x-\frac{\pi}{6}\right)^3+\ldots$ Applies Taylor's correctly about $\frac{\pi}{6}$ with their values/numerical trig expressions. If values are not seen separately the work should imply a correct formula but allow a recognisable attempt at the series following the correct general formula stated. **Requires previous M mark.**		dM1
	$(y=)1+3\left(x-\frac{\pi}{6}\right)+\frac{9}{2}\left(x-\frac{\pi}{6}\right)^2+9\left(x-\frac{\pi}{6}\right)^3+\ldots$	Correct expression with coeffs. in simplest form. "$y=\ldots$" not required. **Requires all previous marks.** Score A0 if **clear evidence** of **use** of any wrong derivative expression.	A1
	If e.g. $y'''\left(\frac{\pi}{6}\right)$ is found by calculator but $y'(x)$ and $y''(x)$ were seen award 1100110 max		**(7)**
	Note: With responses that work in sin and cos throughout, to score M marks there must be no loss of form when differentiating (sign and coefficient errors only, also allowing sign errors with product/quotient formulae). Any use of identities must be correct. E.g: $y=\tan\left(\frac{3x}{2}\right)=\frac{\sin\left(\frac{3x}{2}\right)}{\cos\left(\frac{3x}{2}\right)}\Rightarrow y'=\frac{\frac{3}{2}\cos^2\left(\frac{3x}{2}\right)+\frac{3}{2}\sin^2\left(\frac{3x}{2}\right)}{\cos^2\left(\frac{3x}{2}\right)}$ $y''=\frac{\frac{9}{2}\cos^3\left(\frac{3x}{2}\right)\sin\left(\frac{3x}{2}\right)+\frac{9}{2}\cos\left(\frac{3x}{2}\right)\sin^3\left(\frac{3x}{2}\right)}{\cos^4\left(\frac{3x}{2}\right)}$ or $\frac{\frac{9}{2}\cos\left(\frac{3x}{2}\right)\sin\left(\frac{3x}{2}\right)}{\cos^4\left(\frac{3x}{2}\right)}$ or $\frac{9\sin\left(\frac{3x}{2}\right)}{2\cos^3\left(\frac{3x}{2}\right)}$		
	$y'''=\frac{\frac{27}{4}\cos^8\left(\frac{3x}{2}\right)+27\cos^6\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)+\frac{81}{4}\cos^4\left(\frac{3x}{2}\right)\sin^4\left(\frac{3x}{2}\right)}{\cos^8\left(\frac{3x}{2}\right)}=\frac{27}{4}+27\tan^2\left(\frac{3x}{2}\right)+\frac{81}{4}\tan^4\left(\frac{3x}{2}\right)$		

Question Number	Scheme	Notes	Marks
4(b)	$\left\{y\left(\frac{\pi}{4}\right)=\right\}1+3\left(\frac{\pi}{4}-\frac{\pi}{6}\right)+\frac{9}{2}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)^2+9\left(\frac{\pi}{4}-\frac{\pi}{6}\right)^3$ or $1+3\left(\frac{\pi}{12}\right)+\frac{9}{2}\left(\frac{\pi}{12}\right)^2+9\left(\frac{\pi}{12}\right)^3$ Substitutes $\frac{\pi}{4}$ into their expression for y of the correct form with at least the first three terms (series about $\frac{\pi}{6}$). Must have values (not unevaluated trig expressions). If only a decimal value is given then it must be the correct awrt 2.26 to score M1 (2.255314325). If there is no working they must obtain an expression with at least $a+b\pi+c\pi^2$ and correct exact ft a, b and c for their series or $1+\frac{\pi}{4}+c\pi^2$ with correct exact ft c		M1
	$=1+\frac{\pi}{4}+\frac{\pi^2}{32}+\frac{\pi^3}{192}$ or $1+\frac{1}{4}\pi+\frac{1}{32}\pi^2+\frac{1}{192}\pi^3$	Correct answer or values for A (32) and B (192). Can be awarded if full marks were not scored in (a).	A1
			(2)
			Total 9

Question Number	Scheme	Marks
6(a)	$y=\sec x \Rightarrow \dfrac{dy}{dx}=\sec x\tan x$ $\dfrac{d^2y}{dx^2}=(\sec x\tan x)\tan x+\sec x\left(\sec^2 x\right)\ \left(=\sec x\tan^2 x+\sec^3 x\right)$	M1
	$\dfrac{d^2y}{dx^2}=\sec x\left(\sec^2 x-1\right)+\sec^3 x=2\sec^3 x-\sec x$ $\Rightarrow \dfrac{d^3y}{dx^3}=6\sec^2 x(\sec x\tan x)-\sec x\tan x$ **or** $\dfrac{d^3y}{dx^3}=2\sec x\tan x\sec^2 x+\tan^2 x\sec x\tan x+3\sec^2 x\sec x\tan x$	**d**M1
	$\left(\dfrac{d^3y}{dx^3}=5\sec^3 x\tan x+\sec x\tan x\left(\sec^2 x-1\right)=6\sec^3 x\tan x-\sec x\tan x\right)$ $\Rightarrow \dfrac{d^3y}{dx^3}=\sec x\tan x\left(6\sec^2 x-1\right)$	**dd**M1 A1
		(4)
(b)	$\sec\left(\dfrac{\pi}{3}\right)=2,\ \tan\left(\dfrac{\pi}{3}\right)=\sqrt{3}\Rightarrow f\left(\dfrac{\pi}{3}\right)=2,\ f'\left(\dfrac{\pi}{3}\right)=2\sqrt{3},\ f''\left(\dfrac{\pi}{3}\right)=14,\ f'''\left(\dfrac{\pi}{3}\right)=46\sqrt{3}$	M1
	$f(x)=f(a)+(x-a)f'(a)+\dfrac{(x-a)^2}{2!}f''(a)+\dfrac{(x-a)^3}{3!}f'''(a)+\ldots$ $\Rightarrow 2+2\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+7\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{23\sqrt{3}}{3}\left(x-\dfrac{\pi}{3}\right)^3$	**d**M1 A1
		(3)
(c)	$\left(\sec\dfrac{7\pi}{24}\approx\right)=2+2\sqrt{3}\left(-\dfrac{\pi}{24}\right)+7\left(-\dfrac{\pi}{24}\right)^2+\dfrac{23\sqrt{3}}{3}\left(-\dfrac{\pi}{24}\right)^3=\ldots$	M1
	$\sec\dfrac{7\pi}{24}\approx 1.636709263\ldots\approx 1.637$	A1
		(2)
		Total 9