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8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 8y = 4\ln x \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \qquad \text{(II)}$$

(6)

(b) Determine the general solution of differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of differential equation (I).

(1)

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2. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{1}{z+1} \qquad z \neq -1$$

The imaginary axis in the z-plane is mapped by T onto the curve C in the w-plane.

Show that C is a circle and find its centre and radius.

(6)

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8. (a) Given that $t = \ln x$, where $x > 0$, show that

$$\frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

(3)

(b) Hence show that the transformation $t = \ln x$, where $x > 0$, transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 2y = 1 + 4\ln x - 2(\ln x)^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 1 + 4t - 2t^2 \qquad \text{(II)}$$

(1)

(c) Solve differential equation (II) to determine y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

9: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv a\sin^5\theta + b\sin^3\theta + c\sin\theta$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the possible exact values of $\sin^2\left(\frac{k\pi}{5}\right)$ where $k \in \mathbb{Z}$

(4)

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6. (a) Show that the transformation $x = e^t$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = e^{2t} \qquad \text{(II)}$$

(6)

(b) Find the general solution of the differential equation (II), expressing y as a function of t.

(6)

(c) Hence find the general solution of the differential equation (I).

(1)

DO NOT WRITE IN THIS AREA

7. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$x\frac{dy}{dx} + y + 4x^2y^3 \ln x = 0 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - \frac{2z}{x} = 8x\ln x \qquad x > 0 \qquad \text{(II)}$$

(5)

(b) By solving differential equation (II), determine the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(6)

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2. (a) Find the general solution of the differential equation

$$\left(x^2 + 1\right)\frac{dy}{dx} + xy - x = 0$$

giving your answer in the form $y = f(x)$.

(6)

(b) Find the particular solution for which $y = 2$ when $x = 3$

(2)

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1. Solve the equation

$$z^5 = 32$$

Give your answers in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$

(5)

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8. (a) Show that the substitution $x = e^t$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 13y = 0, \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0$$

(7)

(b) Hence find the general solution of the differential equation (I).

(5)

6. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 8e^{-3t} \qquad t \geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Determine the general solution of this differential equation.

(6)

Given that the motion of the particle satisfies $x = \frac{1}{2}$ and $\frac{dx}{dt} = \frac{1}{2}$ when $t = 0$

(b) determine the particular solution for the motion of the particle.

(4)

On the graph of the particular solution found in part (b), the first turning point for $t > 0$ occurs at $x = a$.

(c) Determine, to 3 significant figures, the value of a.

[*Solutions relying entirely on calculator technology are not acceptable.*]

(4)

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1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express the complex number

$$-4-4\sqrt{3}\mathrm{i}$$

in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, where $r>0$ and $-\pi<\theta\leqslant\pi$

(3)

(b) Solve the equation

$$z^3+4+4\sqrt{3}\mathrm{i}=0$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leqslant\pi$

(4)