2. (a) Show that the differential equation

$$(x+2)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 6x - y \qquad x \neq -2$$

can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{f}(x)y = kx$$

where f is a function to be determined and k is a constant to be found.

(2)

Given that $y = 18$ at $x = 4$

(b) use the answer to part (a) to determine, in simplest form, the particular solution of the differential equation.
Give the answer in the form $y = \mathrm{g}(x)$

(6)

Leave blank

6. Find the general solution of the differential equation

$$\cos x\frac{\mathrm{d}y}{\mathrm{d}x} + y\sin x = (\cos^2 x)\ln x, \qquad 0 < x < \frac{\pi}{2}$$

Give your answer in the form $y = \mathrm{f}(x)$.

(8)

4. (a) Show that the substitution $y^2 = \frac{1}{t}$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y = xy^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}t}{\mathrm{d}x} - 2t = -2x \qquad \text{(II)}$$

(3)

(b) Solve differential equation (II) and determine y^2 in terms of x.

(6)

3. (a) Show that the transformation $y = \frac{1}{z}$ transforms the differential equation

$$x^2 \frac{dy}{dx} + xy = 2y^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{2}{x^2} \qquad \text{(II)}$$

(3)

(b) Solve differential equation (II) to determine z in terms of x.

(4)

(c) Hence determine the particular solution of differential equation (I) for which $y = -\frac{3}{8}$ at $x = 3$

Give your answer in the form $y = f(x)$.

(2)

4. (a) Determine the general solution of the differential equation

$$\frac{dy}{dx} - 3y\tan x = e^{4x}\sec^3 x$$

giving your answer in the form $y = f(x)$

(5)

(b) Determine the particular solution for which $y = 4$ at $x = 0$

(2)

Leave blank

3. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = xe^{-x^2}y^3 \quad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 4xz = -2xe^{-x^2} \quad \text{(II)}$$

(4)

(b) Solve differential equation (II) to find z as a function of x.

(5)

(c) Hence find the general solution of differential equation (I), giving your answer in the form $y^2 = \mathrm{f}(x)$.

(1)

Leave blank

2. (a) Find the general solution of the differential equation

$$\left(x^2 + 1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + xy - x = 0$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

(b) Find the particular solution for which $y = 2$ when $x = 3$

(2)

DO NOT WRITE IN THIS AREA

6: (a) Show that the substitution $z = \dfrac{1}{y^2}$ transforms the differential equation

$$2\frac{dy}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0 \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - (\cot x)z = \tan x \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(3)

(b) Hence determine the general solution of differential equation (I), giving your answer in the form $y^2 = \text{f}(x)$

(5)

Given that $y^2 = \dfrac{4\sqrt{3}}{3}$ when $x = \dfrac{\pi}{6}$

(c) determine the exact values of y when $x = \dfrac{\pi}{3}$

(3)

8. (a) For all the values of x where the identity is defined, prove that

$$\cot 2x + \tan x \equiv \operatorname{cosec} 2x$$

(3)

(b) Show that the substitution $y^2 = w \sin 2x$, where w is a function of x, transforms the differential equation

$$y\frac{dy}{dx} + y^2 \tan x = \sin x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{dw}{dx} + 2w \operatorname{cosec} 2x = \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(4)

(c) By solving differential equation (II), determine a general solution of differential equation (I) in the form $y^2 = \text{f}(x)$, where f(x) is a function in terms of $\cos x$

$$\left[\text{You may use without proof } \int \operatorname{cosec} 2x \, dx = \frac{1}{2}\ln|\tan x| \ (+\text{constant})\right]$$

(6)

4. (a) Show that the substitution $y^2 = \frac{1}{z}$ transforms the differential equation

$$\frac{dy}{dx} + 2y = 3xy^3 \qquad y \neq 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4z = -6x \qquad \text{(II)}$$

(3)

(b) Obtain the general solution of differential equation (II).

(5)

(c) Hence obtain the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(1)

8. (a) Show that the transformation $v = y - 2x$ transforms the differential equation

$$\frac{dy}{dx} + 2yx(y - 4x) = 2 - 8x^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} = -2xv^2 \qquad \text{(II)}$$

(4)

(b) Solve the differential equation (II) to determine v as a function of x

(4)

(c) Hence obtain the general solution of the differential equation (I).

(1)

(d) Sketch the solution curve that passes through the point $(-1, -1)$.

On your sketch show clearly the equation of any horizontal or vertical asymptotes.

You do **not** need to find the coordinates of any intercepts with the coordinate axes or the coordinates of any stationary points.

(5)