Leave blank

5.

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

(a) Show that

$$\frac{d^4y}{dx^4} = \left(ax^2 + b\right)\frac{d^2y}{dx^2}$$

where a and b are constants to be found.

(5)

Given that $y = 1$ and $\frac{dy}{dx} = 3$ at $x = 0$

(b) find a series solution for y in ascending powers of x up to and including the term in x^4

(5)

(c) use your series to estimate the value of y at $x = -0.2$, giving your answer to four decimal places.

(2)

7. Given that $y = e^x \sin x$

(a) show that

$$\frac{d^6y}{dx^6} = k\frac{d^2y}{dx^2}$$

where k is a constant to be determined.

(4)

(b) Hence determine the first 5 non-zero terms in the Maclaurin series expansion for y, giving each coefficient in simplest form.

(3)

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4. Given that

$$y\frac{d^2y}{dx^2} - 4\left(\frac{dy}{dx}\right)^2 + 3y = 0$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{28}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{24}{y}\left(\frac{dy}{dx}\right)$$

(5)

Given also that $y = 8$ and $\frac{dy}{dx} = 1$ at $x = 0$

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3, simplifying the coefficients where possible.

(4)

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5. $$y = e^{\cos^2 x}$$

(a) Show that

$$\frac{d^2y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$$

(4)

(b) Hence find the Maclaurin series expansion of $e^{\cos^2 x}$ up to and including the term in x^2

(3)

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4. $$f(x) = \sin\left(\frac{3}{2}x\right)$$

(a) Find the Taylor series expansion for $f(x)$ about $\frac{\pi}{3}$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^4$

(6)

(b) Hence obtain an estimate of $\sin\frac{1}{2}$, giving your answer to 4 decimal places.

(2)

DO NOT WRITE IN THIS AREA

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that when $y = \arcsin 2x$

$$\frac{dy}{dx} = 2\left(1 - 4x^2\right)^{-\frac{1}{2}}$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{Ax^2 + 8}{\left(1 - 4x^2\right)^{\frac{5}{2}}}$$

where A is a constant to be determined.

(3)

(b) Hence determine the Maclaurin series expansion for arcsin $2x$ in ascending powers of x up to and including the term in x^3

(2)

The Maclaurin series expansion for e^x is given by

$$e^x = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^r}{r!} + \ldots$$

(c) Use the Maclaurin series expansion for e^{3x} and the answer to part (b) to show that, for small values of x

$$e^{3x}\arcsin 2x \approx Cx + Dx^2 + Ex^3$$

where C, D and E are constants to be determined.

(3)

3.

$$2x\frac{d^2y}{dx^2} + y\frac{dy}{dx} - 6xy = 0$$

Given that $\frac{dy}{dx} = 3$ and $y = \frac{1}{2}$ at $x = 2$

(a) determine the value of $\frac{d^3y}{dx^3}$ at $x = 2$

(6)

(b) Hence determine the series expansion for y about $x = 2$, in ascending powers of $(x - 2)$ up to and including the term in $(x - 2)^3$, giving each coefficient in simplest form.

(2)

5. Given that

$$y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2-2y=0 \qquad y>0$$

(a) determine $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y

(4)

Given that $y=2$ and $\frac{dy}{dx}=1$ at $x=0$

(b) determine a series solution for y in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

(4)

Leave blank

5. Given that $y=\tan^2 x$

(a) show that

$$\frac{d^3y}{dx^3}=8\tan x\sec^2 x(p\sec^2 x+q)$$

where p and q are integers to be determined.

(5)

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\tan^2 x$ in ascending powers of $\left(x-\frac{\pi}{3}\right)$ up to and including the term in $\left(x-\frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

(3)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Determine, in ascending powers of $\left(x-\frac{\pi}{6}\right)$ up to and including the term in $\left(x-\frac{\pi}{6}\right)^3$, the Taylor series expansion about $\frac{\pi}{6}$ of

$$y=\tan\left(\frac{3x}{2}\right)$$

giving each coefficient in simplest form.

(7)

(b) Hence show that

$$\tan\frac{3\pi}{8}\approx 1+\frac{\pi}{4}+\frac{\pi^2}{A}+\frac{\pi^3}{B}$$

where A and B are integers to be determined.

(2)

6. Given that $y=\sec x$

(a) show that

$$\frac{d^3y}{dx^3}=\sec x\tan x\left(p\sec^2 x+q\right)$$

where p and q are integers to be determined.

(4)

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\sec x$ in ascending powers of $\left(x-\frac{\pi}{3}\right)$, up to and including the term in $\left(x-\frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

(3)

(c) Use the answer to part (b) to determine, to four significant figures, an approximate value of $\sec\left(\frac{7\pi}{24}\right)$

(2)