Leave blank

4. A transformation from the z-plane to the w-plane is given by

$$w = \frac{z}{z+3}, \qquad z \neq -3$$

Under this transformation, the circle $|z| = 2$ in the z-plane is mapped onto a circle C in the w-plane.

Determine the centre and the radius of the circle C.

(7)

Leave blank

2. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{1}{z+1} \qquad z \neq -1$$

The imaginary axis in the z-plane is mapped by T onto the curve C in the w-plane.

Show that C is a circle and find its centre and radius.

(6)

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7. The point P represents a complex number z on an Argand diagram, where

$$|z+1| = |2z-1|$$

and the point Q represents a complex number w on the Argand diagram, where

$$|w| = |w-1+\mathrm{i}|$$

Find the exact coordinates of the points where the locus of P intersects the locus of Q.

(7)

Leave blank

8. (a) Use de Moivre's theorem to show that

$$\cos^5\theta \equiv p\cos 5\theta + q\cos 3\theta + r\cos\theta$$

where p, q and r are rational numbers to be found.

(6)

(b) Hence, showing all your working, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^5\theta \, \mathrm{d}\theta$$

(4)

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7. A transformation from the z-plane to the w-plane is given by

$$w = \frac{(1+\mathrm{i})z + 2(1-\mathrm{i})}{z - \mathrm{i}} \qquad z \neq \mathrm{i}$$

The transformation maps points on the imaginary axis in the z-plane onto a line in the w-plane.

(a) Find an equation for this line.

(2)

The transformation maps points on the real axis in the z-plane onto a circle in the w-plane.

(b) Find the centre and radius of this circle.

(6)

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5. The transformation T from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$ is given by

$$w = \frac{z+1}{z-3} \qquad z \neq 3$$

The straight line in the z-plane with equation $y = 4x$ is mapped by T onto the circle C in the w-plane.

(a) Show that C has equation

$$3u^2 + 3v^2 - 2u + v + k = 0$$

where k is a constant to be determined.

(5)

(b) Hence determine

(i) the coordinates of the centre of C

(ii) the radius of C

(2)

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DO

3: A complex number z is represented by the point P on an Argand diagram where $|z| = 1$

(a) Sketch the locus of P as z varies.

(1)

The transformation T from the z–plane, where $z = x + iy$, to the w–plane, where $w = u + iv$, is given by

$$w = \frac{9iz - i}{z+1} \qquad z \neq -1$$

Given that the image under T of the locus of P in the z–plane, where $z \neq -1$, is the line l in the w–plane,

(b) determine, in simplest form, a Cartesian equation for l

(5)

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3. A transformation maps points from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$. The transformation is given by

$$w = \frac{(2+i)z + 4}{z - i} \qquad z \neq i$$

The transformation maps the imaginary axis in the z-plane onto the line l in the w-plane.

Determine a Cartesian equation of l, giving your answer in the form $au + bv + c = 0$ where a, b and c are integers to be found.

(6)

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8. Given that $z = e^{i\theta}$

(a) show that $z^n + \frac{1}{z^n} = 2\cos n\theta$

where n is a positive integer.

(2)

(b) Show that

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$$

(5)

(c) Hence solve the equation

$$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta = 0 \qquad 0 \leqslant \theta \leqslant \pi$$

Give your answers to 3 significant figures.

(4)

(d) Use calculus to determine the exact value of

$$\int_0^{\frac{\pi}{3}} (32\cos^6\theta - 4\cos^2\theta)\,d\theta$$

Solutions relying entirely on calculator technology are not acceptable.

(5)

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