8. (a) For all the values of x where the identity is defined, prove that

$$\cot 2x + \tan x \equiv \operatorname{cosec} 2x$$

(3)

(b) Show that the substitution $y^2 = w \sin 2x$, where w is a function of x, transforms the differential equation

$$y\frac{dy}{dx} + y^2 \tan x = \sin x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{dw}{dx} + 2w \operatorname{cosec} 2x = \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(4)

(c) By solving differential equation (II), determine a general solution of differential equation (I) in the form $y^2 = f(x)$, where $f(x)$ is a function in terms of $\cos x$

$$\left[\text{You may use without proof } \int \operatorname{cosec} 2x \, dx = \frac{1}{2}\ln\left|\tan x\right| \ (+\text{constant})\right]$$

(6)

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4. $$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$$

is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 40e^{-x}\sin 3x$$

where A is a constant.

(a) Find the value of A.

(5)

(b) Hence find the general solution of this differential equation.

(4)

(c) Find the particular solution of this differential equation for which both $y = 3$ and $\frac{dy}{dx} = 3$ at $x = 0$

(4)

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6. Obtain the general solution of the equation

$$x^2 \frac{dy}{dx} + (x\cot x + 2)xy = 4\sin x \qquad 0 < x < \pi$$

Give your answer in the form $y = f(x)$

(8)

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8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 8y = 4\ln x \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \qquad \text{(II)}$$

(6)

(b) Determine the general solution of differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of differential equation (I).

(1)

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 2x^2 + x$$

(8)

(b) Find the particular solution of this differential equation for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

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7. (a) Show that the substitution $y = vx$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 - x^2)y = 2x^3 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2v}{dx^2} - v = 2 \qquad \text{(II)}$$

(5)

(b) By solving differential equation (II), find the general solution of differential equation (I) in the form $y = f(x)$.

(6)

Given that $y = e$ and $\frac{dy}{dx} = e$ at $x = 1$

(c) find the particular solution of differential equation (I).

(4)

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7. (a) Show that the substitution $x = e^u$, where u is a function of x, transforms the differential equation

$$2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 27x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$2\frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} \qquad \text{(II)}$$

(4)

(b) By solving differential equation (II), determine the general solution of differential equation (I).

Give the answer in the form $y = f(x)$ where f is a fully simplified function.

(4)

Given that when $x = \frac{1}{4}$, $y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$

(c) determine the value of y when $x = \frac{1}{8}$, giving the answer in the form $\frac{1}{64}\left(p\sqrt{2} + q\right)$ where p and q are integers.

(5)

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2\sin x \quad \text{(I)}$$

(8)

Given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(b) find the particular solution of differential equation (I).

(5)

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8. (a) Show that the transformation $v = y - 2x$ transforms the differential equation

$$\frac{dy}{dx} + 2yx(y - 4x) = 2 - 8x^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} = -2xv^2 \qquad \text{(II)}$$

(4)

(b) Solve the differential equation (II) to determine v as a function of x

(4)

(c) Hence obtain the general solution of the differential equation (I).

(1)

(d) Sketch the solution curve that passes through the point $(-1, -1)$.

On your sketch show clearly the equation of any horizontal or vertical asymptotes.

You do **not** need to find the coordinates of any intercepts with the coordinate axes or the coordinates of any stationary points.

(5)

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6.

$$x\frac{dy}{dx} + (1 - 2x)y = x, \qquad x > 0$$

Find the general solution of the differential equation, giving your answer in the form $y = f(x)$.

(9)