Question Number	Scheme		Marks
3.(a)	$\{\frac{1}{2}(e^x+e^{-x})\}^2-\{\frac{1}{2}(e^x-e^{-x})\}^2=\{\frac{1}{4}(e^{2x}+2+e^{-2x})\}-\{\frac{1}{4}(e^{2x}-2+e^{-2x})\}$		M1
	M1: Uses the correct exponential forms for cosh and sinh and squares both brackets obtaining 3 terms each time		
	$\frac{1}{2}+\frac{1}{2}=1$	**At least one line of intermediate working** (e.g. combines fractions with a common denominator) with no errors seen and concludes = 1	A1
			(2)
(b)	$(e^x-e^{-x})+7\times\frac{1}{2}(e^x+e^{-x})=9$ $\Rightarrow\frac{9}{2}e^x+\frac{5}{2}e^{-x}-9=0$	M1: Uses exponential forms **and** collects terms A1: Any correct form with terms collected	M1A1
	$\Rightarrow 9e^{2x}-18e^x+5=0$ so $e^x=\ldots$	Solves their three term quadratic in e^x as far as $e^x=$	M1
	$e^x=\frac{1}{3}$ or $\frac{5}{3}$	Both values correct	A1
	$x=\ln\frac{1}{3}$ and $\ln\frac{5}{3}$	Both values correct (accept equivalents)	A1
			(5)
			Total 7
	Alternatives for (b) – Special Cases		
Way 2	$2\sinh x=9-7\cosh x\Rightarrow 45\cosh^2 x-126\cosh x+85=0$		M1A1
	M1: Attempt to square both sides A1: Correct quadratic in cosh*x*		
	$(15\cosh x-17)(3\cosh x-5)=0\Rightarrow\cosh x=\frac{17}{15}$ or $\cosh x=\frac{5}{3}$		
	$\frac{e^x+e^{-x}}{2}=\frac{17}{15}\Rightarrow 15e^{2x}-34e^x+15=0,\ \frac{e^x+e^{-x}}{2}=\frac{5}{3}\Rightarrow 3e^{2x}-10e^x+3=0$		
	$(5e^x-3)(3e^x-5)=0\Rightarrow e^x=\frac{3}{5},\ e^x=\frac{5}{3}$ $(3e^x-1)(e^x-3)=0\Rightarrow e^x=\frac{1}{3},\ e^x=3$ $e^x=\frac{5}{3}$ and $e^x=\frac{1}{3}$	M1: Solves at least one of their three term quadratics in e^x as far as $e^x=\ldots$, having used the correct exponential form for cosh*x* A1: $e^x=\frac{5}{3}$ and $e^x=\frac{1}{3}$ seen	M1A1
	$x=\ln\frac{1}{3}$ and $\ln\frac{5}{3}$	These values only with $\ln 3$ and $\ln\frac{3}{5}$ rejected	A1

1	$7\cosh x+3\sinh x=2e^x+7\Rightarrow$ $7\left(\frac{e^x+e^{-x}}{2}\right)+3\left(\frac{e^x-e^{-x}}{2}\right)=2e^x+7$ $\{\frac{7}{2}e^x+\frac{7}{2}e^{-x}+\frac{3}{2}e^x-\frac{3}{2}e^{-x}=2e^x+7\}$	Substitutes at least one correct exponential form for either of the hyperbolic terms and achieves an **equation** in exponentials and constants alone	M1
	$\Rightarrow 7(e^{2x}+1)+3(e^{2x}-1)=4e^{2x}+14e^x$ $\{\Rightarrow 5e^{2x}+2=2e^{2x}+7e^x\}$	Multiplies through by e^x to obtain any equation that would form a 3TQ in e^x if like terms were collected	M1
	$\Rightarrow 6e^{2x}-14e^x+4=0\quad\{3e^{2x}-7e^x+2=0\}$	A correct three term quadratic in e^x. Could be implied by a correct root even if terms have not been collected.	A1
	$\Rightarrow(3e^x-1)(e^x-2)=0\Rightarrow e^x=\ldots$	Solves their 3TQ - usual rules. One correct root for their quadratic if no working. Ignore labelling of the roots even if e.g.,"x" is used.	M1
	$x=\ln 2,\ \ln\frac{1}{3}$	Both correct and simplified but do not isw if there are **other answers.** Allow $-\ln\frac{1}{2}$ for $\ln 2$ and $-\ln 3$ or $\ln 3^{-1}$ for $\ln\frac{1}{3}$	A1
	Answer only is 0/5		**Total 5**

3 **Way 1** **Converts to sinh and cosh**	$4\tanh x - \operatorname{sech} x = 1$ $4\frac{\sinh x}{\cosh x} - \frac{1}{\cosh x} = 1$ $4\sinh x - 1 - \cosh x = 0$ $4\frac{e^x - e^{-x}}{2} - 1 - \frac{e^x + e^{-x}}{2} = 0$	Replaces **one** hyperbolic function with its correct exponential equivalent. Allow for correct replacement of just e.g., $\sinh x$ after using $\tanh x = \frac{\sinh x}{\cosh x}$ May follow errors but do not allow any further marks if the original equation was reduced to one in a single hyperbolic function.	M1
	$3e^{2x} - 2e^x - 5 = 0$	M1: Obtains an equation which if terms are collected is a 3TQ (or 2TQ with no constant) in e^x A1: Correct 3TQ	M1 A1
	$e^x = \frac{2 \pm \sqrt{4+60}}{6} \left(\Rightarrow \frac{2+8}{6} = \frac{5}{3}\right)$	M1: Solves 3TQ (or 2TQ with no constant) in e^x. Apply usual rules. If no working seen they must achieve one correct root of their equation to 3sf which may be complex. If 2TQ must get a correct non-zero root of their equation. A1: Any correct unsimplified expression for e^x that includes the positive root. Must be exact	M1 A1
	$x = \ln\frac{5}{3}$	$\ln\frac{5}{3}$, $\ln 1\frac{2}{3}$, $\ln 1.\dot{6}$ only but allow $k = ...$ No unrejected extra solutions	A1
			Total 6

Question Number	Scheme	Notes	Marks
1(a)	$1 - \tanh^2 x \equiv \operatorname{sech}^2 x$		
	$1 - \tanh^2 x = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$	Replaces the tanh x on the lhs with a **correct** expression in terms of exponentials.	B1
	$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} = \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2}$ or e.g. $\frac{2e^{2x} \times 2e^{-2x}}{(e^x + e^{-x})^2}$ Attempts to find common denominator and expand numerator		M1
	$= \left(\frac{4}{(e^x + e^{-x})^2}\right) = \operatorname{sech}^2 x$ *	Obtains the rhs with no errors.	A1cso
			(3)
ALT 1	$1 - \tanh^2 x = (1 - \tanh x)(1 + \tanh x)$ $= \left(1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)\right)\left(1 + \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)\right)$	Uses the difference of 2 squares on the lhs and replaces the tanh x with a **correct** expression in terms of exponentials.	B1
	$= \left(\frac{2e^{-x}}{e^x + e^{-x}}\right)\left(\frac{2e^x}{e^x + e^{-x}}\right)$	Attempt to find common denominators and simplify numerators.	M1
	$= \left(\frac{4}{(e^x + e^{-x})^2}\right) = \operatorname{sech}^2 x$ *	Obtains the rhs with no errors.	A1cso
ALT 2	$\operatorname{sech}^2 x = \frac{4}{(e^x + e^{-x})^2}$	Replaces the sech x on the rhs with a **correct** expression in terms of exponentials.	B1
	$= \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2} = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$ Attempts to express the "4" in terms of the denominator.		M1
	$= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 = 1 - \tanh^2 x$ *	Obtains the lhs with no errors.	A1cso

	Uses $\text{sech}^2 x = 1 - \tanh^2 x$ and forms a 3 term quadratic in $\tanh x$		
	$(2\tanh x - 1)(\tanh x - 1) = 0 \Rightarrow \tanh x = ...$	Solves 3TQ by any valid method including calculator.	M1
	$\tanh x = \frac{1}{2} \to x = \ln\sqrt{3}$	$\ln\sqrt{3}$. Accept $\frac{1}{2}\ln 3, -\frac{1}{2}\ln\frac{1}{3}$ And no other answers.	A1
			(3)
ALT	$2\text{sech}^2 x + 3\tanh x = 3 \Rightarrow 2\left(\frac{4}{(e^x + e^{-x})^2}\right) + 3\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = 3$ $\Rightarrow 8 + 3(e^{2x} - e^{-2x}) = 3(e^{2x} + 2 + e^{-2x}) \Rightarrow ...$ Substitutes the correct exponential forms, attempts to eliminate fractions and collect terms		M1
	$6e^{-2x} = 2 \Rightarrow e^{-2x} = \frac{1}{3}$	Rearranges to reach $e^{-2x} = ...$	M1
	$x = \ln\sqrt{3}$	$\ln\sqrt{3}$. Accept $\frac{1}{2}\ln 3, -\frac{1}{2}\ln\frac{1}{3}$ And no other answers.	A1
			Total 6

Question Number	Scheme	Marks
	There is no credit for attempts that do not use exponential definitions	
1(a) Working from LHS to RHS	$(2\cosh 5x \cosh x) = 2\frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2} = \frac{e^{6x} + e^{4x} + e^{-4x} + e^{-6x}}{2}$	M1
	$= \frac{e^{6x} + e^{-6x}}{2} + \frac{e^{4x} + e^{-4x}}{2} = \cosh 6x + \cosh 4x$ *	A1*
		(2)
ALT Working from RHS to LHS	$\cosh 6x + \cosh 4x = \frac{e^{6x} + e^{-6x}}{2} + \frac{e^{4x} + e^{-4x}}{2} = \frac{e^{6x} + e^{4x} + e^{-4x} + e^{-6x}}{2} = 2\frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2}$	M1
	$= 2\frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2} = 2\cosh 5x \cosh x$	A1*
(b)	$\cosh 6x + \cosh 4x = 8\cosh x \Rightarrow 2\cosh 5x \cosh x = 8\cosh x$ $\Rightarrow 2\cosh x(\cosh 5x - 4) = 0 \Rightarrow \cosh 5x = ...$ or $2\cosh 5x = ...$	M1
	$\cosh 5x = 4$ or $2\cosh 5x = 8$	A1
	$\cosh 5x = 4 \Rightarrow 5x = \ln\left(4 + \sqrt{4^2 - 1}\right)$ or $\cosh 5x = 4 \Rightarrow \frac{e^{5x} + e^{-5x}}{2} = 4 \Rightarrow e^{10x} - 8e^{5x} + 1 = 0 \Rightarrow e^{5x} = \frac{8 \pm \sqrt{60}}{2}$	M1
	$x = \pm\frac{1}{5}\ln\left(4 + \sqrt{15}\right)$ or $x = \frac{1}{5}\ln\left(4 \pm \sqrt{15}\right)$	A1
		(4)
		Total 6