Question Number	Scheme	Notes	Marks
2(a)	$\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}2-\lambda & 0 & 3\\ 0 & -4-\lambda & -3\\ 0 & -4 & -\lambda\end{vmatrix}$ $=$ e.g., $(2-\lambda)\left[(-4-\lambda)(-\lambda)-(-4)(-3)\right]-0+3(0)$ or $(2-\lambda)\left[(-4-\lambda)(-\lambda)-(-4)(-3)\right]-0+0$ Sarrus $\Rightarrow (2-\lambda)(-4-\lambda)(-\lambda)-(2-\lambda)(-3)(-4)$	Obtains an unsimplified cubic expression for $\det(\mathbf{M}-\lambda\mathbf{I})$ condoning sign/copying slips only. Allow poor bracketing if intention clear.	**M1**
	Note: It is possible to just use $\mathbf{M}\boldsymbol{x}=\lambda\boldsymbol{x}$ e.g., $-4y=\lambda z\Rightarrow y=-\frac{\lambda z}{4}$ and $-4y-3z=\lambda y\Rightarrow \lambda z-3z=-\frac{\lambda^2 z}{4}\Rightarrow \lambda^2+4\lambda-12=0\Rightarrow ...$ Score the M1 for achieving a 3TQ in λ from appropriate work condoning copying/sign slips only		
	$(2-\lambda)(\lambda^2+4\lambda-12)=0$ or $\lambda^3+2\lambda^2-20\lambda+24=0$ or $-\lambda^3-2\lambda^2+20\lambda-24=0$ $(2-\lambda)(\lambda-2)(\lambda+6)=0$ or $(\lambda+6)(\lambda-2)(\lambda-2)=0$ $\lambda_1=-6\ \ (\lambda_2=2)$ dM1: Solves $\det(\mathbf{M}-\lambda\mathbf{I})=0$ to obtain any value for λ including 2. Not usual rules – award for any value seen that is consistent with their equation. The "=0" can be implied by a solution. Note that they may disregard the $(2-\lambda)$ and solve a quadratic. A1: -6 from a correct equation. Accept both solutions e.g., "-6, 2" and allow if mislabelled and/or -6 rejected. No incorrect solutions.		**dM1** **A1**
	$\mathbf{M}x=-6x\ \Rightarrow\ \begin{matrix}2x+3z=-6x\\ -4y-3z=-6y\\ -4y=-6z\end{matrix}\ \Rightarrow x=...,\ y=...,\ z=...$ $(\mathbf{M}+6\mathbf{I})x=0\ \Rightarrow\ \begin{matrix}8x+3z=0\\ 2y-3z=0\\ -4y+6z=0\end{matrix}\ \Rightarrow x=...,\ y=...,\ z=...$	Uses $\mathbf{M}\boldsymbol{x}=\lambda\boldsymbol{x}$ or $(\mathbf{M}-\lambda\mathbf{I})\boldsymbol{x}=0$ with any of their non-zero eigenvalues (however obtained) to form simultaneous equations and solves. No requirement for a vector for this mark. There is no need to check their values but award M0 for a zero solution.	**M1**
	Note: Could find vector product of first 2 rows of $\mathbf{M}-\lambda\mathbf{I}$ i.e., $(8\mathbf{i}+3\mathbf{k})\times(2\mathbf{j}-3\mathbf{k})=(-6\mathbf{i}+24\mathbf{j}+16\mathbf{k})$ (two correct components)		
	$\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-3\\12\\8\end{pmatrix}\Rightarrow\frac{1}{\sqrt{3^2+12^2+8^2}}\begin{pmatrix}-3\\12\\8\end{pmatrix}$	Correct method to normalise their eigenvector no matter how this vector is obtained provided it has at least 2 non-zero components. Only allow slips if there is working.	**M1**
	e.g., $\frac{1}{\sqrt{217}}\begin{pmatrix}-3\\12\\8\end{pmatrix}$ or $\begin{pmatrix}-\frac{3\sqrt{217}}{217}\\ \frac{12\sqrt{217}}{217}\\ \frac{8\sqrt{217}}{217}\end{pmatrix}$ or $\begin{pmatrix}-\frac{3}{\sqrt{217}}\\ \frac{12}{\sqrt{217}}\\ \frac{8}{\sqrt{217}}\end{pmatrix}$ or $\frac{1}{2\sqrt{217}}\begin{pmatrix}-6\\24\\16\end{pmatrix}$	A correct normalised eigenvector in any form. Note direction may be reversed. May use **i**, **j**, **k** notation	**A1**
			(6)

Question Number	Scheme	Notes	Marks
2(b)	May use $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation		
	Multiplies position and direction by $\mathbf{M}$ (not e.g., $\mathbf{M}-\lambda\mathbf{I}$) In parametric form: $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}4+2\mu\\-1\\-\mu\end{pmatrix}=\ldots \quad \left\{\begin{pmatrix}8+4\mu-3\mu\\4+3\mu\\4\end{pmatrix}=\begin{pmatrix}8\\4\\4\end{pmatrix}+\mu\begin{pmatrix}1\\3\\0\end{pmatrix}\right\}$ There is no requirement to extract the vectors if parametric form is used. Allow this mark if e.g., $8+4\mu-3\mu$ written as $2(4+2\mu)-3\mu$ Allow this work without a parameter i.e., $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}4\\-1\\0\end{pmatrix}=\ldots \left\{\begin{pmatrix}8\\4\\4\end{pmatrix}\right\}$ **and** $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}2\\0\\-1\end{pmatrix}=\ldots \left\{\begin{pmatrix}1\\3\\0\end{pmatrix}\right\}$ **or** $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}4&2\\-1&0\\0&-1\end{pmatrix}=\ldots \left\{\begin{pmatrix}8&1\\4&3\\4&0\end{pmatrix}\right\}$ Alternatively: Could find 2 points on l_1, transform them both and **subtract** to find direction. Allow slips and condone the matrix product written the wrong way round provided they have attempted to multiply the elements appropriately and they obtain a vector (or 3 x 2 matrix) with the resulting values correctly placed. Condone if they proceed to confuse which is the position and which is the direction.		M1
	$\mathbf{r}\times\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}8\\4\\4\end{pmatrix}\times\begin{pmatrix}1\\3\\0\end{pmatrix}$	Forms: $\mathbf{r}\times\text{direction}=\text{position}\times\text{direction}$ Must not clearly confuse their vectors. Allow if RHS = direction x position. **Requires previous M mark.** No requirement to calculate vector product but the RHS could be implied by 2 correct components (or the negative version if the product is reversed)	dM1
	$\mathbf{r}\times\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}-12\\4\\20\end{pmatrix}$	Any correct equation in the correct form. Not $\mathbf{b}=\ldots,\ \mathbf{c}=\ldots$ unless $\mathbf{r}\times\mathbf{b}=\mathbf{c}$ seen. Isw once a correct answer is seen.	A1
			(3)
			Total 9

Question Number	Scheme		Marks
4. (a)	$\det \mathbf{M} = 6 - k^2$	A correct (possibly un-simplified) determinant	B1
	$\mathbf{M}^T = \begin{pmatrix} 3 & k & k \\ k & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **or** minors $\begin{pmatrix} 2 & k & -2k \\ k & 3 & -k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$ **or** cofactors $\begin{pmatrix} 2 & -k & -2k \\ -k & 3 & k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$		B1
	$\frac{1}{6-k^2}\begin{pmatrix} 2 & -k & 0 \\ -k & 3 & 0 \\ -2k & k^2 & 6-k^2 \end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1A1A1
			(5)
(b)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ -2 & 1 & 5 \end{pmatrix}\begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix}$ $\Rightarrow a = \ldots \text{or } b = \ldots \text{or } c = \ldots$	Uses $k = 1$ in the inverse and attempts to multiply to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao
			(3)
			Total 8
	Alternative for (b)		
	$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix} \Rightarrow \begin{matrix} 3a + b = -5 \\ a + 2b = 10 \\ a + c = 7 \end{matrix}$ $\Rightarrow a = \ldots \text{or } b = \ldots \text{or } c = \ldots$	Multiplies to give 3 equations and attempts to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao

Question Number	Scheme	Notes	Marks
3(a)	3	Correct value seen in (a)	B1
			(1)
(b)	$\begin{pmatrix}-2 & 5 & 0\\ 5 & 1 & -3\\ 0 & -3 & 6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}8x\\ 8y\\ 8z\end{pmatrix}\Rightarrow \begin{matrix}-2x+5y=8x\\ 5x+y-3z=8y\\ -3y+6z=8z\end{matrix}\Rightarrow\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\ldots$ Correct method for the eigenvector (making a variable equal to 0 is not a correct method)		M1
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1\\ 2\\ -3\end{pmatrix}$	Any correct eigenvector	A1
			(2)
(c)	$\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\begin{vmatrix}-2-\lambda & 5 & 0\\ 5 & 1-\lambda & -3\\ 0 & -3 & 6-\lambda\end{vmatrix}=0$ $\Rightarrow(-2-\lambda)\left[(1-\lambda)(6-\lambda)-9\right]-5\left[5(6-\lambda)\right]=0\Rightarrow\lambda=\ldots$ NB CE is $\lambda^3-5\lambda^2-42\lambda+144=0$ but may only find the constant term		M1
	$\lambda=-6$	Correct third eigenvalue The work for these 2 marks may be seen in (a) – award them Correct third eigenvalue by a different method – send to review	A1
	$\mathbf{D}=\begin{pmatrix}3 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & -6\end{pmatrix}$	Correct **D** following through their third eigenvalue	A1ft
	$\begin{pmatrix}-2 & 5 & 0\\ 5 & 1 & -3\\ 0 & -3 & 6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}-6x\\ -6y\\ -6z\end{pmatrix}\Rightarrow \begin{matrix}-2x+5y=-6x\\ 5x+y-3z=-6y\\ -3y+6z=-6z\end{matrix}\Rightarrow\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\left(\begin{pmatrix}-5\\ 4\\ 1\end{pmatrix}\right)$ Correct strategy for 3rd eigenvector		M1
	$\mathbf{P}=\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}}\\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}}\\ \frac{1}{\sqrt{3}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{42}}\end{pmatrix}$	Fully correct matrix consistent with their **D** May have $\frac{\sqrt{3}}{3}$ etc	A1
			(5)
			Total 8

Question Number	Scheme	Notes	Marks
3(a)	$\begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix} = \lambda\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$	Correct statement	M1
	$7-3=\lambda$ or $28=7\lambda \Rightarrow \lambda = 4$	Correct eigenvalue	A1
			(2)
(b)	$7+4\times19+k=4\times19 \Rightarrow k=-7$ *	M1: Uses y component to establish an equation for k A1*: Correct k	M1A1*
			(2)
(c)	$\begin{vmatrix} 0-\lambda & 1 & 9 \\ 1 & 4-\lambda & -7 \\ 1 & 0 & -3-\lambda \end{vmatrix} = 0$		
	$\lambda(4-\lambda)(3+\lambda)+(3+\lambda)-7+9(\lambda-4)=0$ or $-7+9(\lambda-4)-(3+\lambda)[\lambda(\lambda-4)-1]$	M1: Correct characteristic equation method (allow sign errors only) A1: Correct equation in any form	M1A1
	$(4-\lambda)[\lambda(3+\lambda)-1-9]=0$	NB $\lambda^3-\lambda^2-22\lambda+40=0$	
	$(\lambda-2)(\lambda+5)=0 \Rightarrow \lambda = 2, -5$	A1: $\lambda=2$ or $\lambda=-5$ A1: $\lambda=2$ and $\lambda=-5$	A1A1
			(4)
(d) **Way 1**	$\begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & -7 \\ 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} q+9r \\ p+4q-7r \\ p-3r \end{pmatrix}$	Multiplies by **M** to obtain a vector in terms of p, q and r	M1
	$\begin{pmatrix} q+9r \\ p+4q-7r \\ p-3r \end{pmatrix} = \begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$	Correct equations	A1
	$p=2, q=3, r=-1$	M1: Solves simultaneously to obtain at least one of p, q or r. **Dependent on the previous method mark.** A1: Correct answers	dM1A1
	Correct equations followed by correct answers scores full marks in part (d)		
			(4)

Question	Scheme	Notes	Marks
2	Condone poor notation e.g., determinant lines used for matrix bracketing		
(a)	$\det\begin{pmatrix}2&0&0\\0&1&4\\3&-2&-3\end{pmatrix}\left\{=2\times(-3+8)\right\}=10$	Correct value for determinant, seen or stated and not just in a final answer	B1
	$\left\{\text{Minors}:\begin{pmatrix}5&-12&-3\\0&-6&-4\\0&8&2\end{pmatrix}\Rightarrow\right\}\ \text{Cofactors}:\begin{pmatrix}5&12&-3\\0&-6&4\\0&-8&2\end{pmatrix}$	Attempts the cofactor matrix with at least 6 correct elements	M1
	Inverse is $\frac{1}{\text{"10"}}\begin{pmatrix}5&0&0\\12&-6&-8\\-3&4&2\end{pmatrix}$ or e.g., $\begin{pmatrix}\frac{1}{2}&0&0\\\frac{6}{5}&-\frac{3}{5}&-\frac{4}{5}\\-\frac{3}{10}&\frac{2}{5}&\frac{1}{5}\end{pmatrix}$	Correct inverse but allow ft on their "10". Allow equivalent fractions/decimals. A0 if clearly obtained incorrectly	A1ft
	Work to obtain Adj(**M**) must be seen but it may be minimal, e.g., sight of the matrix of minors followed by the correct answer is acceptable. Note that B0 M1 A1 is possible.		**(3)**
(b)	$\frac{1}{10}\begin{pmatrix}5&0&0\\12&-6&-8\\-3&4&2\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\ldots$	Multiplies their $\mathbf{M}^{-1}$ by $\begin{pmatrix}u\\v\\w\end{pmatrix}$ Must use a matrix other than **M** – not just changed by application of determinant. Condone sight of $\mathbf{vM}^{-1}=\ldots$ but must not be a clearly incorrect multiplication method	M1
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{10}\begin{pmatrix}5u\\12u-6v-8w\\-3u+4v+2w\end{pmatrix}$ or $\begin{pmatrix}\frac{1}{2}u\\\frac{6}{5}u-\frac{3}{5}v-\frac{4}{5}w\\-\frac{3}{10}u+\frac{2}{5}v+\frac{1}{5}w\end{pmatrix}$ or $\frac{1}{d}\begin{pmatrix}5u\\12u-6v-8w\\-3u+4v+2w\end{pmatrix}$ or $\begin{pmatrix}\frac{5}{d}u\\\frac{12}{d}u-\frac{6}{d}v-\frac{8}{d}w\\-\frac{3}{d}u+\frac{4}{d}v+\frac{2}{d}w\end{pmatrix}$ A1ft: Two correct vector components, coordinates or equations, ft their $d \neq 0$ A1ft: All three correct ft their non-zero $d \neq 0$ Must be exact (and not rounded decimals for ft) **These ft marks are not available for an incorrect** Adj(**M**)		A1ft A1ft
			(3)

Question	Scheme	Notes	Marks
2(c)	$3x-7y+2z=-3\Rightarrow 3\left(\frac{1}{2}u\right)-7\left(\frac{6}{5}u-\frac{3}{5}v-\frac{4}{5}w\right)+2\left(-\frac{3}{10}u+\frac{2}{5}v+\frac{1}{5}w\right)=-3$	Substitutes their expressions into the equation for Π_1	M1
	$-15u+10v+12w=-6$	Correct **equation.** Terms in any order but constant isolated. Accept any integer multiples.	A1
			(2)
			Total 8