Question Number	Scheme	Notes	Marks
3(a)	$\begin{pmatrix}0&1&9\\1&4&k\\1&0&-3\end{pmatrix}\begin{pmatrix}7\\19\\1\end{pmatrix}=\lambda\begin{pmatrix}7\\19\\1\end{pmatrix}$	Correct statement	M1
	$7-3=\lambda$ or $28=7\lambda \Rightarrow \lambda=4$	Correct eigenvalue	A1
			(2)
(b)	$7+4\times19+k=4\times19 \Rightarrow k=-7$ *	M1: Uses *y* component to establish an equation for *k* A1*: Correct *k*	M1A1*
			(2)
(c)	$\begin{vmatrix}0-\lambda&1&9\\1&4-\lambda&-7\\1&0&-3-\lambda\end{vmatrix}=0$		
	$\lambda(4-\lambda)(3+\lambda)+(3+\lambda)-7+9(\lambda-4)=0$ or $-7+9(\lambda-4)-(3+\lambda)[\lambda(\lambda-4)-1]$	M1: Correct characteristic equation method (allow sign errors only) A1: Correct equation in any form	M1A1
	$(4-\lambda)[\lambda(3+\lambda)-1-9]=0$	NB $\lambda^3-\lambda^2-22\lambda+40=0$	
	$(\lambda-2)(\lambda+5)=0 \Rightarrow \lambda=2,-5$	A1: $\lambda=2$ or $\lambda=-5$ A1: $\lambda=2$ and $\lambda=-5$	A1A1
			(4)
(d) Way 1	$\begin{pmatrix}0&1&9\\1&4&-7\\1&0&-3\end{pmatrix}\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}q+9r\\p+4q-7r\\p-3r\end{pmatrix}$	Multiplies by **M** to obtain a vector in terms of *p*, *q* and *r*	M1
	$\begin{pmatrix}q+9r\\p+4q-7r\\p-3r\end{pmatrix}=\begin{pmatrix}-6\\21\\5\end{pmatrix}$	Correct equations	A1
	$p=2, q=3, r=-1$	M1: Solves simultaneously to obtain at least one of *p*, *q* or *r*. **Dependent on the previous method mark.** A1: Correct answers	dM1A1
	Correct equations followed by correct answers scores full marks in part (d)		
			(4)

Question Number	Scheme		Marks
4. (a)	$\det \mathbf{M} = 6 - k^2$	A correct (possibly un-simplified) determinant	B1
	$\mathbf{M}^T = \begin{pmatrix} 3 & k & k \\ k & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **or** minors $\begin{pmatrix} 2 & k & -2k \\ k & 3 & -k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$ **or** cofactors $\begin{pmatrix} 2 & -k & -2k \\ -k & 3 & k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$		B1
	$\frac{1}{6-k^2}\begin{pmatrix} 2 & -k & 0 \\ -k & 3 & 0 \\ -2k & k^2 & 6-k^2 \end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1A1A1
			(5)
(b)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ -2 & 1 & 5 \end{pmatrix}\begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix}$ $\Rightarrow a = \ldots \text{or } b = \ldots \text{or } c = \ldots$	Uses $k = 1$ in the inverse and attempts to multiply to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao
			(3)
			Total 8

	Alternative for (b)		
	$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix} \Rightarrow \begin{matrix} 3a + b = -5 \\ a + 2b = 10 \\ a + c = 7 \end{matrix}$ $\Rightarrow a = \ldots \text{or } b = \ldots \text{or } c = \ldots$	Multiplies to give 3 equations and attempts to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao

Question	Scheme	Notes	Marks
4	$\mathbf{M}=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$		
(a)	$\begin{vmatrix}1-\lambda & 1 & 3\\ 1 & 5-\lambda & 1\\ 3 & 1 & 1-\lambda\end{vmatrix}(=0)$ $\Rightarrow(1-\lambda)\{(5-\lambda)(1-\lambda)-1\}-(1-\lambda-3)+3(1-3(5-\lambda))\ \ (=0)$ M1: Attempt characteristic equation (at least 2 'elements' correct) $\left[\text{"Elements" are } (1-\lambda)\{(5-\lambda)(1-\lambda)-1\},\ -(1-\lambda-3),\ +3(1-3(5-\lambda))\right]$		M1
	$\lambda=6\Rightarrow-5\times4+8+12=0$ or $\lambda^3-7\lambda^2+36=(\lambda-6)(\lambda^2-\lambda-6)\Rightarrow\lambda=6$	Verifies $\lambda=6$ is an eigenvalue or factorises cubic to $(\lambda-6)\times$quadratic and extracts $\lambda=6$	A1
	$(\lambda^2-\lambda-6)=0\Rightarrow\lambda=3,-2$	M1: Solves their 3 term quadratic or cubic $(\lambda-6)(\lambda^2+k\lambda\pm6)$ seen A1: Two other correct eigenvalues	M1A1
			(4)
ALT	Sub $\lambda=6$ into $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert$ and shows this = 0 with no further work scores M1A1M0A0 For a "factor theorem" solution (ie sub further values for λ), one further correct value scores M1A1M1A0. Both further correct values scores 4/4		
	Solutions without working: (calculator?) M1A1 as above; M1A1 correct values or M0A0 one or both incorrect		

Question	Scheme	Notes	Marks
(b)	$\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=6\begin{pmatrix}x\\ y\\ z\end{pmatrix}$ or $\begin{pmatrix}-5 & 1 & 3\\ 1 & -1 & 1\\ 3 & 1 & -5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$ M1: Either statement is sufficient or equivalent in equation form		M1
	$x+y+3z=6x, x+5y+z=6y, 3x+y+z=6z$ $\Rightarrow x=\ldots$ or $y=\ldots$ or $z=\ldots$	Solves two equations to obtain one variable in terms of another	dM1
	$\begin{pmatrix}1\\ 2\\ 1\end{pmatrix}$ or $x=k,\ y=2k,\ z=k\ \ k\neq0$	Any multiple	A1
	$\pm\begin{pmatrix}\frac{1}{\sqrt{6}}\\ \frac{2}{\sqrt{6}}\\ \frac{1}{\sqrt{6}}\end{pmatrix}$ oe	Correct normalised vector Can be positive or negative Can be written in the **i**,**j**,**k** form	A1 **(4)**
			Total 8

Question Number	Scheme	Marks
6(a)	$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix}$	
	$\lvert\mathbf{A}\rvert = 12 - 2 - k(20+12) + 2(-5-18)$ (via first row) $\lvert\mathbf{A}\rvert = 12 - 2 - 5(4k+2) + 6(-2k-6)$ (via first column) $\lvert\mathbf{A}\rvert = 12 - 12k - 10 - 36 - 2 - 20k$ (Sarrus)	M1
	$\lvert\mathbf{A}\rvert = 0 \Rightarrow 10 - 32k - 46 = 0 \Rightarrow k = \ldots$	M1
	$k = -\dfrac{9}{8}$	A1
		(3)
(b)	$\begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \to \begin{pmatrix} 10 & 32 & -23 \\ 4k+2 & -8 & -1-6k \\ -2k-6 & -12 & 3-5k \end{pmatrix} \to \begin{pmatrix} 10 & -32 & -23 \\ -4k-2 & -8 & 6k+1 \\ -2k-6 & 12 & 3-5k \end{pmatrix}$	M1A1
	$\begin{pmatrix} 10 & -32 & -23 \\ -4k-2 & -8 & 6k+1 \\ -2k-6 & 12 & 3-5k \end{pmatrix} \to \begin{pmatrix} 10 & -4k-2 & -2k-6 \\ -32 & -8 & 12 \\ -23 & 6k+1 & 3-5k \end{pmatrix}$ $\mathbf{A}^{-1} = \dfrac{-1}{32k+36}\begin{pmatrix} 10 & -4k-2 & -2k-6 \\ -32 & -8 & 12 \\ -23 & 6k+1 & 3-5k \end{pmatrix}$	dM1A1
		(4)
		Total 7

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A}=\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 2 & a\end{pmatrix}$		
	$\lvert\mathbf{A}\rvert = a-2+a-1+2-1(=2a-2)$	Correct determinant in any form	B1
	$\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 2 & a\end{pmatrix}\to\begin{pmatrix}a-2 & a-1 & 1\\ -a-2 & a-1 & 3\\ -2 & 0 & 2\end{pmatrix}\to\begin{pmatrix}a-2 & 1-a & 1\\ a+2 & a-1 & -3\\ -2 & 0 & 2\end{pmatrix}$ Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed		M1
	$\begin{pmatrix}a-2 & 1-a & 1\\ a+2 & a-1 & -3\\ -2 & 0 & 2\end{pmatrix}\to\begin{pmatrix}a-2 & a+2 & -2\\ 1-a & a-1 & 0\\ 1 & -3 & 2\end{pmatrix}$ Correct transpose of cofactors		A1
	$\mathbf{A}^{-1}=\frac{1}{2a-2}\begin{pmatrix}a-2 & a+2 & -2\\ 1-a & a-1 & 0\\ 1 & -3 & 2\end{pmatrix}$	Correct inverse	A1
			(4)
(b)	$a=4\Rightarrow\mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$=\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}12-6\lambda\\ 4+2\lambda\\ 6+3\lambda\end{pmatrix}=\ldots$	Attempt to multiply the parametric form of l_2 by their inverse	M1
	$=\begin{pmatrix}6-\lambda\\ -4+4\lambda\\ 2-\lambda\end{pmatrix}$	Correct parametric form	A1
	$\mathbf{r}=\begin{pmatrix}6\\ -4\\ 2\end{pmatrix}+\lambda\begin{pmatrix}-1\\ 4\\ -1\end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l=\ldots$ award A0	A1
			(4)
			Total 8