Question Number	Scheme	Notes	Marks
3(a)(i)	$(\pm 7e, 0)$	Correct **coordinates** or $x = \pm 7e,\ y = 0$	B1
(ii)	$x = \pm\dfrac{7}{e}$	Correct **equations**	B1
	SC: If "49" used for "7" consistently in (i) and (ii) score B0 B1		
			(2)
(b)(i)	$\left(PS^2 =\right)(x - '7e')^2 + y^2$ oe e.g. $\left(PS^2 =\right)('7e' - x)^2 + y^2$	Correct expression or equivalent with their $7e$. Must be in terms of e, x and y only. Apply isw once a correct expression is seen.	B1ft
(ii)	$\left(PM^2 =\right)\left('\dfrac{7}{e}' - x\right)^2$ oe e.g. $\left(x - '\dfrac{7}{e}'\right)^2$	Correct expression or equivalent with their $\dfrac{7}{e}$. Must be in terms of e and x only. Apply isw once a correct expression is seen.	B1ft
			(2)
(c)	$\left(\dfrac{PS}{PM} = e \Rightarrow\right) PS^2 = e^2PM^2 \Rightarrow (x - '7e')^2 + y^2 = e^2\left('\dfrac{7}{e}' - x\right)^2$ $\Rightarrow x^2 - 14ex + 49e^2 + y^2 = 49 - 14ex + e^2x^2$ Applies $PS^2 = e^2PM^2$ with their PS and PM and expands (condone poor squaring)		M1
	$x^2(1-e^2) + y^2 = 49(1-e^2)$ $\Rightarrow \dfrac{x^2}{49} + \dfrac{y^2}{49(1-e^2)} = 1 \Rightarrow b^2 = 49(1-e^2)$*	Reaches given answer with fully correct proof. All shown steps required. Note that it is possible to obtain this result even if the B marks are not scored in (b) e.g. correct expressions but not in the forms required.	A1*
			(2)
(d)	$(4\sqrt{3})^2 = 49(1-e^2) \Rightarrow e^2 \ldots$ or $e = \ldots$	Replaces b^2 with $(4\sqrt{3})^2$ and solves for e^2 or e.	M1
	$e = \dfrac{1}{7}$	Correct exact value for e (Not $\pm$)	A1
			(2)
(e)	$x = \dfrac{7}{2} \Rightarrow \dfrac{\left(\frac{7}{2}\right)^2}{49} + \dfrac{y^2}{'48'} = 1 \Rightarrow y = \ldots[(\pm)6]$	Substitutes into their ellipse equation and obtains a value for y	M1
	Area $\Delta OPM = \left(\dfrac{1}{2}\right)\left(\dfrac{7}{'\left(\frac{1}{7}\right)'} - \dfrac{7}{2}\right)('6') = \ldots$ Correct method for area of triangle OPM with their $\dfrac{7}{e}$ and their 6 May see other approaches, e.g., "shoelace" method e.g. $\dfrac{1}{2}\begin{vmatrix} 3.5 & 0 & 49 & 3.5 \\ 6 & 0 & 6 & 6 \end{vmatrix} = \dfrac{1}{2}(49\times 6 - 6\times 3.5) = \ldots$		dM1
	$\dfrac{273}{2}$ or $136\dfrac{1}{2}$ or 136.5	Any correct exact value	A1
	Special Case: $x = \dfrac{7}{2} \Rightarrow \dfrac{\left(\frac{7}{2}\right)^2}{49} + \dfrac{y^2}{'48'} = 1 \Rightarrow y = 36 \Rightarrow$ Area $\Delta OPM = \left(\dfrac{1}{2}\right)\left(\dfrac{7}{'\left(\frac{1}{7}\right)'} - \dfrac{7}{2}\right)(36) = \ldots(819)$ **Scores M0M1A0**		
			(3)
			Total 11

Question Number	Scheme		Marks
2.	$\pm\frac{a}{e}=\pm 9$ **and** $a^2(1-e^2)=8$	Both equations correct	B1
	$a^4-81a^2+648=0$ or $81e^4-81e^2+8=0$	M1: Eliminates an unknown to produce a quadratic in a^2 or e^2 A1: Correct three term quadratic in any form with terms collected	M1A1
	$(a^2-72)(a^2-9)=0\Rightarrow a^2=...$ or $(9e^2-8)(9e^2-1)=0\Rightarrow e^2=...$	Uses a standard method (see notes) to solve quadratic as far as $a^2=...$ or $e^2=...$ (Must be $a^2=...$ or $e^2=...$ at this stage not $a=...$ or $e=....$ but this may be implied by later work) **May be implied by correct answers only.**	M1
	$a=3$ and $a=6\sqrt{2}$	M1: Complete method to find a. Either square roots from $a^2=...$ or square roots from $e^2=....$ and uses $a=9e$ at least once A1: cao (both answers correct). Do not accept ± for either of the answers unless the negative is rejected later.	M1A1
			(6)
			Total 6

Question Number	Scheme	Notes	Marks

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6(a)	Normal to plane given by $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{vmatrix} = \ldots$	Attempt cross product of direction vectors. If the method is unclear, look for at least 2 correct components.	M1
	$= 6\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	Or any multiple of this vector.	A1
	Substitute appropriate point into $6x + 2y - 2z = d$ e.g. (1, 1, 1) or (2, 1, 4) to find "d"	Use a valid point and use scalar product with normal or substitute into Cartesian equation.	M1
	$6x + 2y - 2z = 6$ $3x + y - z = 3$ *	Given answer. No errors seen	A1* cso
			(4)
(a) ALT	$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\Rightarrow x = 1 + \lambda + \mu,\ y = 1 - 2\mu,\ z = 1 + 3\lambda + \mu$ M1: Forms equation of plane using (1, 1, 1) and direction vectors and extracts 3 equations for x, y and z in terms of λ and μ A1: Correct equations		M1A1
	$x = 1 + \frac{1}{2} - \frac{1}{2}y + \frac{1}{3}z - \frac{1}{2} + \frac{1}{6}y$	Eliminates λ and μ and achieves an equation in x, y and z only.	M1
	$3x + y - z = 3$ *	Given answer. No errors seen.	A1
(b)	$s = -3$	cao	B1
			(1)
(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -2 \\ 3 & 1 & -1 \end{vmatrix} = \mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$	Attempts cross product of normal vectors. If the method is unclear, look for at least 2 correct components.	M1
	e.g. $x = 0, 2y - 2z = 6, y - 2z = 3$ $\Rightarrow y = 3, z = 0$	Any valid attempt to find a point on the line.	M1
	e.g. $(0,3,0)$	Any valid point on the line	A1
	$\mathbf{r} = 3\mathbf{j} + \lambda(\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$	Correct equation including "**r** =" or equivalent e.g. $x = \frac{y-3}{-5} = \frac{z}{-2}$	A1
			(4)

(d)	$(3\mathbf{i}+\mathbf{j}-\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}-2\mathbf{k})=6$	Correct value for scalar product	B1
		Full scalar product attempt to reach a value for $\cos\theta$	M1

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	$\cos\theta=\dfrac{(3\mathbf{i}+\mathbf{j}-\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})}{\sqrt{9+1+1}\sqrt{1+1+4}}=\sqrt{\dfrac{6}{11}}$	For $\cos\theta=\sqrt{\dfrac{6}{11}}$	A1
	$\theta=42.4°$	Correct value. Mark their final answer.	A1
			(4)
(d) ALT	$\lvert(3\mathbf{i}+\mathbf{j}-\mathbf{k})\times(\mathbf{i}+\mathbf{j}-2\mathbf{k})\rvert=\sqrt{30}$	Correct value for magnitude of cross product	B1
	$\sin\theta=\dfrac{\lvert(3\mathbf{i}+\mathbf{j}-\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})\rvert}{\sqrt{9+1+1}\sqrt{1+1+4}}=\dfrac{\sqrt{55}}{11}$	Full attempt to reach a value for $\sin\theta$	M1
		For $\sin\theta=\dfrac{\sqrt{55}}{11}$	A1
	$\theta=42.4°$	Correct value. Mark their final answer.	A1
			Total 13

Question Number	Scheme		Marks
2 (a)(i)	$2\cosh^2 x-1=2\dfrac{\left(e^{x}+e^{-x}\right)^2}{4}-1=\dfrac{\left(e^{2x}+2e^{x}\times e^{-x}+e^{-2x}\right)}{2}-1$ Substitutes the correct definition for cosh*x* into the rhs and squares - full expansion must be seen but allow 2 for $2e^{x}\times e^{-x}$		M1
	$=\dfrac{\left(e^{2x}+e^{-2x}\right)}{2}+1-1=\cosh 2x$*	Correct completion with no errors seen.	A1
	Working from left to right:		
	$\cosh 2x=\dfrac{\left(e^{2x}+e^{-2x}\right)}{2}=\dfrac{\left(e^{x}+e^{-x}\right)^2-2}{2}$ Uses the correct definition for cosh2*x* on lhs and expresses in terms of $\left(e^{x}+e^{-x}\right)^2$.		M1
	$2\cosh^2 x-1$*	Correct completion with no errors seen.	A1
(ii)	$2\sinh x\cosh x=2\dfrac{\left(e^{x}-e^{-x}\right)}{2}\times\dfrac{\left(e^{x}+e^{-x}\right)}{2}=\ldots$ Use both correct definitions on rhs and attempts to multiply $2\sinh x\cosh x=\dfrac{1}{2}\left(e^{x}-e^{-x}\right)\left(e^{x}+e^{-x}\right)=\ldots$scores M0 as the definitions for sinh*x* and cosh*x* have not been seen		M1
	$\dfrac{\left(e^{2x}-e^{-2x}\right)}{2}=\sinh 2x$*	Correct completion with no errors seen.	A1
	Working from left to right:		
	$\sinh 2x=\dfrac{\left(e^{2x}-e^{-2x}\right)}{2}=\dfrac{\left(e^{x}+e^{-x}\right)\left(e^{x}-e^{-x}\right)}{2}$ Uses the correct definition for sinh2*x* on lhs and uses the difference of 2 squares.		M1
	$2\sinh x\cosh x$*	Correct completion with no errors seen.	A1

(b)	$2\cosh^2 x - 1 - 7\cosh x + 7 = 0$	Use the identity for cosh2*x*	M1
	$2\cosh^2 x - 7\cosh x + 6 = 0 \Rightarrow (2\cosh x - 3)(\cosh x - 2) = 0 \Rightarrow \cosh x = ...$ Solve their 3TQ in cosh *x* (the usual rules for solving can be applied if necessary)		M1
	$\cosh x = \frac{3}{2}, 2$	Correct answers, both needed	A1
	$\cosh x = \alpha \Rightarrow x = \ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$ **or** $\frac{e^x + e^{-x}}{2} = 2 \Rightarrow e^{2x} - 4e^x + 1 = 0$ or $\frac{e^x + e^{-x}}{2} = \frac{3}{2} \Rightarrow e^{2x} - 3e^x + 1 = 0$ $\Rightarrow e^x = \frac{4 \pm \sqrt{12}}{2}$ or $e^x = \frac{3 \pm \sqrt{5}}{2}$ $\Rightarrow x = \ln...$ Changes at least one arcosh to ln form either using the correct ln form of arcosh or by returning to the correct exponential form of cosh and solving a quadratic in e^x. (Note that returning to exponentials is more likely to give all 4 answers below)		M1
	$x = \ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right), -\ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)\left(\text{or } \ln\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right)\right),$ $\ln\left(2 + \sqrt{3}\right), -\ln\left(2 + \sqrt{3}\right)\left(\text{or } \ln\left(2 - \sqrt{3}\right)\right)$ All 4 correct, must be exact logarithms but can be any equivalent to those shown **with brackets**. Allow unsimplified if necessary and apply isw e.g. allow $\ln\left(\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1}\right)$ for $\ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$		A1
			(5)
			[Total 9]

Question Number	Scheme	Notes	Marks
2	$\frac{x^2}{25}+\frac{y^2}{4}=1, \quad P(5\cos\theta, 2\sin\theta)$		
(a)	$\frac{dx}{d\theta}=-5\sin\theta, \ \frac{dy}{d\theta}=2\cos\theta$ or $\frac{2x}{25}+\frac{2y}{4}\frac{dy}{dx}=0$	Correct derivatives or correct implicit differentiation	B1
	$\frac{dy}{dx}=\frac{2\cos\theta}{-5\sin\theta}$	Divides their derivatives correctly or substitutes and rearranges	M1
	$M_N=\frac{5\sin\theta}{2\cos\theta}$	Correct perpendicular gradient rule	M1
	$y-2\sin\theta=\frac{5\sin\theta}{2\cos\theta}(x-5\cos\theta)$	Correct straight line method (any complete method) **Must** use their gradient of the normal.	M1
	$5x\sin\theta-2y\cos\theta=21\sin\theta\cos\theta$*	cso	A1*
			(5)
(b)	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta-\frac{21}{2}\sin\theta}{2}\right)$ $\left(=\left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta\right)\right)$	Correct mid-point method for at least one coordinate Can be implied by a correct x coordinate	M1
	L cuts x-axis at $\frac{21}{5}\cos\theta$		B1
	Area $OPM = OLP + OLM$ $\frac{1}{2}.\frac{21}{5}\cos\theta.2\sin\theta+\frac{1}{2}.\frac{21}{5}\cos\theta.\frac{17}{4}\sin\theta$	M1: Correct triangle area method using their coordinates A1: Correct expression	M1A1
	$=\frac{105}{16}\sin 2\theta$	Or $6.5625\sin 2\theta$ must be positive	A1(6)
			Total 11