Question Number	Scheme/Notes			Marks
7(a)	$y = \arccos(\operatorname{sech} x)$			
	e.g.:	$\cos y = \operatorname{sech} x \Rightarrow$		
	$\dfrac{dy}{dx} = -\dfrac{(-\operatorname{sech} x \tanh x)}{\sqrt{1-\operatorname{sech}^2 x}}$	$-\sin y \dfrac{dy}{dx} = -\operatorname{sech} x \tanh x$ or, e.g., $-\sin y = -\operatorname{sech} x \tanh x \dfrac{dx}{dy}$	$\cos y = (\cosh x)^{-1} \Rightarrow$ $-\sin y \dfrac{dy}{dx} = -(\cosh x)^{-2} \sinh x$	M1
	Differentiates to obtain an equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ of the correct form e.g. condone coefficient sign errors only.			
	$\dfrac{dy}{dx} = \dfrac{\operatorname{sech} x \tanh x}{\tanh x}$	$\sqrt{1-\operatorname{sech}^2 x}\dfrac{dy}{dx} = \operatorname{sech} x \tanh x$ $\Rightarrow \tanh x \dfrac{dy}{dx} = \operatorname{sech} x \tanh x$	$\sqrt{1-\operatorname{sech}^2 x}\dfrac{dy}{dx} = \dfrac{\sinh x}{\cosh^2 x}$ $\Rightarrow \tanh x \dfrac{dy}{dx} = \dfrac{\sinh x}{\cosh^2 x}$	dM1
	Uses correct identities to obtain an equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ in terms of x only with no roots but accept $\sqrt{\tanh^2 x}$ as "no roots"			
	$\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	$\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	$\dfrac{dy}{dx} = \dfrac{\cosh x}{\sinh x} \cdot \dfrac{\sinh x}{\cosh^2 x}$ $\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	
	$\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	$\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	$\dfrac{dy}{dx} = \dfrac{\cosh x}{\sinh x} \cdot \dfrac{\sinh x}{\cosh^2 x}$ $\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	
	Fully correct proof. An equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ and exactly two different hyperbolic functions with no roots must be seen before the given answer but accept $\sqrt{\tanh^2 x}$ as "no roots" Withhold this mark for any mathematical error e.g., **clear** use of $\dfrac{d}{dx}(\arccos x) = +\dfrac{1}{\sqrt{1-x^2}}$ and $\dfrac{d}{dx}(\operatorname{sech} x) = +\operatorname{sech} x \tanh x$ or e.g. hyperbolic functions written as trig functions or vice versa. Allow slips if they are recovered but clear and consistent errors score A0			A1*
	Note: There may be other methods seen, e.g., using exponentials and "meeting in the middle"			
				(3)

(b)	e.g. $\frac{d}{dx}(\coth x) = -\text{cosech}^2 x$ or $\frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$ or $\frac{-\text{sech}^2 x}{\tanh^2 x}$ or $1-\coth^2 x$ etc. or e.g. $\frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$ or $\frac{2e^{2x}(e^{2x}-1) - 2e^{2x}(e^{2x}+1)}{(e^{2x}-1)^2}$ or $\frac{-4}{(e^x - e^{-x})^2}$ etc. Correct derivative of coth x in any form. Allow recovery if they write e.g. $-\text{cosec}^2 x$ when $-\text{cosech}^2 x$ is clearly implied by subsequent work.	B1
	e.g., $\text{sech}\, x - \text{cosech}^2 x = 0 \Rightarrow \text{sech}\, x = \text{cosech}^2 x \Rightarrow \frac{1}{\cosh x} = \frac{1}{\sinh^2 x} \Rightarrow$ $a\cosh^2 x + b\cosh x + c = 0$ or $a\,\text{sech}^2 x + b\,\text{sech}\, x + c = 0$ **or** $\text{sech}\, x - \text{cosech}^2 x = 0 \Rightarrow \Rightarrow \frac{2}{e^x + e^{-x}} - \left(\frac{2}{e^x - e^{-x}}\right)^2 = 0 \Rightarrow$ $\Rightarrow Ae^{4x} + Be^{3x} + Ce^{2x} + De^x + E = 0$ Sets $f'(x) = 0$ **and** uses correct identities to obtain a 3TQ in cosh x or sech x **or** substitutes the correct exponential forms and obtains a 5 term quartic in e^x	M1
	$\cosh^2 x - \cosh x - 1 = 0$ or $\text{sech}^2 x + \text{sech}\, x - 1 = 0$ oe **or** $\Rightarrow e^{4x} - 2e^{3x} - 2e^{2x} - 2e^x + 1 = 0$ oe Correct quadratic equation or correct quartic equation.	A1
	$\cosh x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \left(= \frac{1+\sqrt{5}}{2}\right)$ or e.g., $\left(\text{sech}\, x + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 = 0 \Rightarrow \text{sech}\, x = \left(\frac{-1+\sqrt{5}}{2}\right)$ Solves quadratic resulting from sech x + their derivative of coth x = 0 Must obtain a **real and exact** value > 1 (or between 0 and 1 if sech used). Apply usual rules. (No need to reject invalid values) If no solving method seen one solution must be consistent with their equation. For the 5 term quartic in e^x progress is unlikely unless they proceed via e.g. $\left(e^{2x} - (1+\sqrt{5})e^x + 1\right)^2 = 0$	dM1
	$x = \text{arcosh}\left(\frac{1+\sqrt{5}}{2}\right) = \ln\left(\frac{1+\sqrt{5}}{2} + \sqrt{\left(\frac{1+\sqrt{5}}{2}\right)^2 - 1}\right)$ or $\frac{e^x + e^{-x}}{2} = \frac{1+\sqrt{5}}{2} \Rightarrow e^{2x} - (1+\sqrt{5})e^x + 1 = 0 \Rightarrow e^x = \frac{1+\sqrt{5}+\sqrt{(1+\sqrt{5})^2 - 4}}{2} \Rightarrow x = ...$ Uses correct logarithmic form or exponentials to find x as a ln of an exact value. Exponential definition must be correct and quadratic solving subject to usual rules or consistent with their equation leading to a value of $e^x > 0$	ddM1
	$\Rightarrow x = \ln\left(\frac{1}{2}(1+\sqrt{5}) + \sqrt{\frac{1}{2}(1+\sqrt{5})}\right)$ or accept $x = \ln\left(\frac{1+\sqrt{5}}{2} + \sqrt{\frac{1+\sqrt{5}}{2}}\right)$ Note that $x = \ln\frac{1}{2}(1+\sqrt{5}) + \sqrt{\frac{1}{2}(1+\sqrt{5})}$ scores A0	A1
		(6)
		Total 9

Correct work in (b) leading to:

$$\cosh^2 x - \cosh x - 1 = 0 \Rightarrow \cosh x = \frac{1+\sqrt{5}}{2}$$

$$x = \text{arcosh}\left(\frac{1+\sqrt{5}}{2}\right) = \ln\left(\frac{1+\sqrt{5}}{2} + \sqrt{\frac{1+\sqrt{5}}{2}}\right)$$

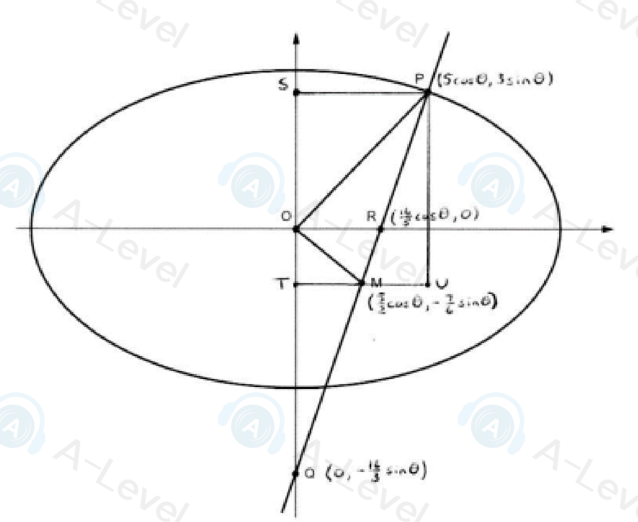
With no evidence where the $\sqrt{\frac{1+\sqrt{5}}{2}}$ comes from, scores: B1M1A1dM1ddM0A0

Question	Scheme	Marks
6(a) Way 1	$I_n=\int_0^{\sqrt{\frac{\pi}{2}}} x^{n-1}.x\cos(x^2)\,dx=\left[x^{n-1}.\frac{1}{2}\sin(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}(n-1)x^{n-2}.\frac{1}{2}\sin(x^2)\,dx$	M1A1
	$=\left[x^{n-1}.\frac{1}{2}\sin(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}-\frac{1}{2}(n-1)\int_0^{\sqrt{\frac{\pi}{2}}}x^{n-3}.x\sin(x^2)\,dx$ $=\left[x^{n-1}.\frac{1}{2}\sin(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}-\frac{1}{2}(n-1)\left(\underline{\left[x^{n-3}.-\frac{1}{2}\cos(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}(n-3)x^{n-4}.-\frac{1}{2}\cos(x^2)\,dx}\right)$	<u>dM1A1</u>
	$=\left(\frac{1}{2}\left(\sqrt{\frac{\pi}{2}}\right)^{n-1}\sin\frac{\pi}{2}-0\right)-\frac{1}{2}(n-1)\left[(0-0)+\frac{1}{2}(n-3)I_{n-4}\right]$	dM1
	$=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}}-\frac{1}{4}(n-1)(n-3)I_{n-4}$ *	A1*
		(6)
Way 2	$I_n=\left[\frac{x^{n+1}}{n+1}.\cos(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}\frac{x^{n+1}}{n+1}.-2x\sin(x^2)\,dx$	M1A1
	$=\left[\frac{x^{n+1}}{n+1}.\cos(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}+\frac{2}{n+1}\int_0^{\sqrt{\frac{\pi}{2}}}x^{n+2}\sin(x^2)\,dx$ $=\left[\frac{x^{n+1}}{n+1}.\cos(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}+\frac{2}{n+1}\left(\underline{\left[\frac{x^{n+3}}{n+3}.\sin(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}-\int_0^{\sqrt{\frac{\pi}{2}}}\frac{x^{n+3}}{n+3}.2x\cos(x^2)\,dx}\right)$	<u>dM1A1</u>
	$=(0-0)+\frac{2}{n+1}\left(\frac{1}{n+3}\left(\sqrt{\frac{\pi}{2}}\right)^{n+3}\sin\frac{\pi}{2}-0-\frac{2}{n+3}I_{n+4}\right)$	dM1
	$\Rightarrow I_{n+4}=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n+3}{2}}-\frac{1}{4}(n+1)(n+3)I_n$ so replacing n by $n-4$ gives $I_n=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}}-\frac{1}{4}(n-1)(n-3)I_{n-4}$ *	A1*
		(6)
(b)	$I_1=\int_0^{\sqrt{\frac{\pi}{2}}}x\cos(x^2)\,dx=\left[\frac{1}{2}\sin(x^2)\right]_0^{\sqrt{\frac{\pi}{2}}}=\frac{1}{2}$	B1
	$I_5=\frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{5-1}{2}}-\frac{1}{4}(5-1)(5-3)\times"\frac{1}{2}"$	M1
	$=\frac{\pi^2}{8}-1$ oe e.g. $\frac{\pi^2-8}{8}$, $\frac{1}{2}\left(\frac{\pi}{2}\right)^2-1$	A1
		(3)

Question Number	Scheme	Notes	Marks
7	$x=\cosh t+t,\quad y=\cosh t-t$		
(a)	$\frac{dx}{dt}=\sinh t+1,\quad \frac{dy}{dt}=\sinh t-1$	Correct derivatives	B1
	$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\sinh^2 t+2\sinh t+1+\sinh^2 t-2\sinh t+1$ $=2\sinh^2 t+2$ M1: Squares correctly, cancels and collects terms		M1
	$=2\left(1+\sinh^2 t\right)=2\cosh^2 t$ *	Uses $\cosh^2 t=1+\sinh^2 t$ to complete the proof with no errors	A1*
			(3)
(b)	$S=2\pi\int y\,ds=2\pi\int(\cosh t-t)\sqrt{2}\cosh t\,dt$	Uses $S=2\pi\int y\,ds$ with the given y and the result from part (a)	M1
	$=2\sqrt{2}\pi\int_0^{\ln 3}\left(\cosh^2 t-t\cosh t\right)dt$ *	Correct proof with no errors	A1*
			(2)
(c)	$\int\cosh^2 t\,dt=\int\pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t\,dt$	Uses $\cosh^2 t=\pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t$	M1
	$\int t\cosh t\,dt=t\sinh t-\int\sinh t\,dt$	Attempts integration by parts the right way round on $t\cosh t$	M1
		Correct expression	A1
	$S=\left(2\sqrt{2}\pi\right)\int\left(\cosh^2 t-t\cosh t\right)dt=\left(2\sqrt{2}\pi\right)\left[\frac{1}{2}t+\frac{1}{4}\sinh 2t-t\sinh t+\cosh t\right]$ A1: 2 correct terms A1: All correct		A1A1
	$(S=)2\sqrt{2}\pi\left\{\left(\frac{1}{2}\ln 3+\frac{10}{9}-\frac{4}{3}\ln 3+\frac{5}{3}\right)-(1)\right\}$ dM1: Correct use of limits 0 and ln3 depends on both preceding M marks		dM1
	$S=\frac{1}{9}\sqrt{2}\pi\left(32-15\ln 3\right)$	cao	A1 **(7)**
			Total 12

Question Number	Scheme	Notes	Marks
6(a)	$\frac{x^2}{25}+\frac{y^2}{9}=1 \quad P(5\cos\theta,\ 3\sin\theta)$		
	$\left\{\frac{dx}{d\theta}=-5\sin\theta \quad \frac{dy}{d\theta}=3\cos\theta\right\}$ $\frac{dy}{dx}=-\frac{3\cos\theta}{5\sin\theta}$; $\frac{2x}{25}+\frac{2y}{9}\frac{dy}{dx}=0$ $\frac{dy}{dx}=-\frac{9x}{25y}\left\{=-\frac{45\cos\theta}{75\sin\theta}\right\}$	$y=\left(9-\frac{9}{25}x^2\right)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{-\frac{18}{25}x}{2\sqrt{9-\frac{9}{25}x^2}}\left\{=\frac{-\frac{18}{25}\times 5\cos\theta}{2\sqrt{9-9\cos^2\theta}}\right\}$	B1
	Any correct expression for $\frac{dy}{dx}$ in terms of θ, or x and y, or x. Allow for a correct $\frac{dx}{dy}$ or $-\frac{dx}{dy}$		
	$m_T=-\frac{3\cos\theta}{5\sin\theta}\Rightarrow m_N=\frac{5\sin\theta}{3\cos\theta}$	Correct perpendicular gradient rule for their $\frac{dy}{dx}$ in terms of θ May see $m_T=-\frac{3}{5}\cot\theta\Rightarrow m_N=\frac{5}{3}\tan\theta$	M1
	$y-3\sin\theta=\frac{5\sin\theta}{3\cos\theta}(x-5\cos\theta)$ OR $y=mx+c\Rightarrow 3\sin\theta=\frac{5\sin\theta}{3\cos\theta}\times 5\cos\theta+c\Rightarrow c=-\frac{16}{3}\sin\theta$ Correct straight line method with a changed gradient in terms of θ		M1
	$3y\cos\theta-9\sin\theta\cos\theta=5x\sin\theta-25\sin\theta\cos\theta$ $\Rightarrow 5x\sin\theta-3y\cos\theta=16\sin\theta\cos\theta$ *	Reaches given answer with intermediate line of working and no errors. Allow this equation written in reverse, x and y terms in different order provided they are together with the third term on the other side and allow the products in a different order provided the numerical coefficients "5", "–3" and "16" are at the front of the terms.	A1*
	The last three marks require $P(5\cos\theta,\ 3\sin\theta)$ to be substituted but condone using e.g. $\frac{25y}{9x}$ as the normal gradient when forming the straight line provided appropriate substitution is seen before the given answer.		
			(4)

Question Number	Scheme	Notes	Marks
6(b)	At Q, $x=0 \Rightarrow y=-\frac{16}{3}\sin\theta$	Correct y coordinate of Q. Accept unsimplified	**B1**
	M is $\left(\dfrac{5\cos\theta+0}{2}, \dfrac{3\sin\theta+'-\frac{16}{3}\sin\theta'}{2}\right)$ Accept $x=\frac{5}{2}\cos\theta,\ y=-\frac{7}{6}\sin\theta$	Correct method for midpoint for both coordinates with their y_Q. Could be implied. **Alternatively**, award for $\Delta OPM=\frac{1}{2}\Delta OPQ=\frac{1}{2}\times\frac{1}{2}\times\frac{16}{3}\sin\theta\times5\cos\theta$ (see area examples below)	**M1**
	e.g., PQ meets x-axis at $R\left(\dfrac{16}{5}\cos\theta,\ 0\right)$ $\Rightarrow$ Area $\Delta OPM=\Delta OPR+\Delta OMR$ $=\dfrac{1}{2}\times\dfrac{16}{5}\cos\theta\left(3\sin\theta+\dfrac{7}{6}\sin\theta\right)$	Correct unsimplified expression for area of ΔOPM Do not allow recovery from a negative area. **Can only follow incorrect work i.e., an incorrect midpoint if $\Delta OPM=\frac{1}{2}\Delta OPQ$ is used.** Please see below for alternatives	**M1**
	If shoelace method is used, score for a correct "extracted" expression for the area (allow with modulus if correct) e.g., $\dfrac{1}{2}\begin{vmatrix}0 & 5\cos\theta & \frac{5}{2}\cos\theta & 0\\ 0 & 3\sin\theta & -\frac{7}{6}\sin\theta & 0\end{vmatrix}$ $\Rightarrow\frac{1}{2}\left\|(5\cos\theta)(-\frac{7}{6}\sin\theta)-(\frac{5}{2}\cos\theta)(3\sin\theta)\right\|$ or $\frac{1}{2}\left[(5\cos\theta)(\frac{7}{6}\sin\theta)+(\frac{5}{2}\cos\theta)(3\sin\theta)\right]$		
	$\left\{=\dfrac{20}{3}\sin\theta\cos\theta=\dfrac{10}{3}\sin2\theta\right\}\Rightarrow(\text{area}=)\dfrac{10}{3}$	Correct area <u>following a correct expression</u>	**A1**
	$\frac{10}{3}$ <u>and justification</u>: **From** $\frac{10}{3}\sin2\theta$: max (value) of $\sin2\theta$ is 1 or e.g., $-1\leqslant\sin2\theta\leqslant1$ or states $\theta=\frac{\pi}{4}$ or 45° or obtains this using differentiation: $\{\frac{10}{3}\}\sin2\theta\Rightarrow\{\frac{20}{3}\}\cos2\theta=0\Rightarrow...$ Do not accept if there is any wrong statement e.g., $\sin2\theta\leqslant1$, $-1<\sin2\theta<1$ but we will condone the ambiguous "$\sin2\theta$ is between 1 and -1" **From any other expression**: Must <u>differentiate</u> (unless rewrites as $\frac{10}{3}\sin2\theta$) e.g., $\frac{20}{3}\sin\theta\cos\theta$ $\frac{20}{3}(\cos^2\theta-\sin^2\theta)\Rightarrow\frac{20}{3}\cos2\theta=0$ or $\tan^2\theta=1\Rightarrow\theta=\frac{\pi}{4}$ or 45° Ignore any further differentiation to justify maximum		**A1**
			(5)
			Total 9



May see:

$\Delta OPM=\frac{1}{2}\Delta OPQ=\frac{1}{2}\times\frac{1}{2}\times\frac{16}{3}\sin\theta\times5\cos\theta$

(Scores the first 2 M marks together since M is not required – ignore an absent or wrong M)

$\Delta OPM=\Delta OPQ-\Delta OMQ$

$\frac{1}{2}\times\frac{16}{3}\sin\theta\times5\cos\theta-\frac{1}{2}\times\frac{16}{3}\sin\theta\times\frac{5}{2}\cos\theta$

$\Delta OPM=\Delta PQS-\Delta OMQ-\Delta PSO$

$=\frac{1}{2}\times(\frac{16}{3}\sin\theta+3\sin\theta)\times5\cos\theta-\frac{1}{2}\times\frac{16}{3}\sin\theta\times\frac{5}{2}\cos\theta-\frac{1}{2}\times3\sin\theta\times5\cos\theta$

$\{=\frac{125}{6}\sin\theta\cos\theta-\frac{20}{3}\sin\theta\cos\theta-\frac{15}{2}\sin\theta\cos\theta\}$

$\Delta OPM=PSTU-\Delta PSO-\Delta OMT-\Delta PMU$

$=5\cos\theta\times(3\sin\theta+\frac{7}{6}\sin\theta)-\frac{1}{2}\times3\sin\theta\times5\cos\theta$

$-\frac{1}{2}\times\frac{5}{2}\cos\theta\times\frac{7}{6}\sin\theta-\frac{1}{2}\times(5\cos\theta-\frac{5}{2}\cos\theta)(3\sin\theta+\frac{7}{6}\sin\theta)$

$\{=(\frac{125}{6}-\frac{15}{2}-\frac{35}{24}-\frac{125}{24})\sin\theta\cos\theta\}$

Note that attempts that start by using Pythagoras for PM will also require the perpendicular distance from O to the line

Question Number	Scheme	Notes	Marks
9	$\frac{x^2}{25}+\frac{y^2}{16}=1, \quad (5\cos\theta, 4\sin\theta)$		
(a)	$\frac{dx}{d\theta}=-5\sin\theta,\ \frac{dy}{d\theta}=4\cos\theta$ **or** $\frac{2x}{25}+\frac{2y}{16}\frac{dy}{dx}=0$ oe **or** $\frac{dy}{dx}=-\frac{4x}{25}\left(1-\frac{x^2}{25}\right)^{-\frac{1}{2}}$ oe	Correct derivatives or correct implicit differentiation or correct explicit differentiation.	B1
	$\frac{dy}{dx}=\frac{4\cos\theta}{-5\sin\theta}$	Divides their derivatives correctly or substitutes and rearranges	M1
	$M_N=\frac{5\sin\theta}{4\cos\theta}$	Correct perpendicular gradient rule – may be implied when they form the normal equation.	M1
	$y-4\sin\theta=\frac{5\sin\theta}{4\cos\theta}(x-5\cos\theta)$	Correct straight line method (any complete method). **Must** use their gradient of the normal.	M1
	$5x\sin\theta-4y\cos\theta=9\sin\theta\cos\theta$* **or** $9\sin\theta\cos\theta=5x\sin\theta-4y\cos\theta$*	Achieves the printed answer with no errors and allow this answer to be obtained from the previous line. Allow $5\sin\theta x$ for $5x\sin\theta$ and $4\cos\theta y$ for $4y\cos\theta$.	A1*
	Allow all marks if the gradient is seen as a function of x and y initially (even in the straight line equation) as long as this is recovered correctly.		
	Solutions that do not use calculus e.g. just quoting the equation of the normal as $y-4\sin\theta=\frac{5\sin\theta}{4\cos\theta}(x-5\cos\theta)$ **send to review however if they just quote e.g.** $ax\sin\theta-by\sin\theta=(a^2-b^2)\sin\theta\cos\theta$ **and then write down the given result this scores no marks. But we would accept** $\frac{dy}{dx}=\frac{4\cos\theta}{-5\sin\theta}$ **to be quoted for a full solution.**		
			(5)
(b)	$b^2=a^2(1-e^2)\Rightarrow 16=25(1-e^2)\Rightarrow e=\frac{3}{5}$ F is $(ae,\ 0)=\left(5\times\frac{3}{5},\ 0\right)$ Or e.g. $"c"^2=a^2e^2=a^2-b^2=25-16\Rightarrow a^2e^2=9\Rightarrow ae=\ldots$ Fully correct strategy for F (must be numerical so $(5e, 0)$ is M0		M1
	(3, 0)	Correct coordinates. (±3, 0) scores A0	A1
			(2)

(c)	$x=\frac{9}{5}\cos\theta$	Correct x coordinate (of Q)	B1
	$PF^2=(5\cos\theta-\text{"3"})^2+(4\sin\theta)^2$ or $PF=\sqrt{(5\cos\theta-\text{"3"})^2+(4\sin\theta)^2}$	Correct application of Pythagoras to find PF or PF^2. Their "3" should be positive but allow work in terms of e e.g. "5e".	M1
	$=25\cos^2\theta-30\cos\theta+9+16\sin^2\theta$ $=25\cos^2\theta-30\cos\theta+9+16(1-\cos^2\theta)$	Applies $\sin^2\theta=1-\cos^2\theta$ to obtain a quadratic expression in $\cos\theta$. If the correct identity is not seen explicitly then their working must imply that a correct identity has been used. **Depends on the previous M.**	dM1
	$PF=\pm(5-3\cos\theta)$ $PF^2=9\cos^2\theta-30\cos\theta+25$	Correct expression for PF or PF^2 in terms of cos θ with terms collected.	A1
	Note that an alternative to using Pythagoras to find PF is to use $PF=ePM$ where M is the foot of the perpendicular from P to the **positive** directrix. Score M1 for $x=\frac{a}{e}=\frac{5}{3/5}\left(=\frac{25}{3}\right)$ (not $\pm\frac{25}{3}$) and **d**M1A1 for $PF=ePM=\frac{3}{5}\left(\frac{25}{3}-5\cos\theta\right)$		
	$\frac{\lvert QF\rvert}{\lvert PF\rvert}=\frac{3-\frac{9}{5}\cos\theta}{5-3\cos\theta}=\frac{3\left(1-\frac{3}{5}\cos\theta\right)}{5\left(1-\frac{3}{5}\cos\theta\right)}$ or e.g. $\frac{3}{5}\times\frac{1-\frac{3}{5}\cos\theta}{1-\frac{3}{5}\cos\theta}=\frac{3}{5}=e*$ **or e.g.** $\frac{QF^2}{PF^2}=\frac{\left(3-\frac{9}{5}\cos\theta\right)^2}{9\cos^2\theta-30\cos\theta+25}=\frac{9-\frac{54}{5}\cos\theta+\frac{81}{25}\cos^2\theta}{9\cos^2\theta-30\cos\theta+25}$ $=\frac{9\left(1-\frac{6}{5}\cos\theta+\frac{9}{25}\cos^2\theta\right)}{25\left(1-\frac{6}{5}\cos\theta+\frac{9}{25}\cos^2\theta\right)}$ or e.g. $=\frac{9}{25}\times\frac{1-\frac{6}{5}\cos\theta+\frac{9}{25}\cos^2\theta}{1-\frac{6}{5}\cos\theta+\frac{9}{25}\cos^2\theta}=\frac{9}{25}\Rightarrow\frac{QF}{PF}=\frac{3}{5}=e*$ Fully correct working including factorisation or equivalent leading to showing that $\frac{\lvert QF\rvert}{\lvert PF\rvert}=e$ with no errors and a conclusion "$=e$". Note that the value of e must have been seen earlier e.g. in part (b) or calculated independently somewhere in the question. Note that this mark depends on a ratio where the numerator and denominator are either both positive or both negative or modulus symbols are present throughout. This does not apply to the second case as both numerator and denominator must be positive as they are squared.		A1*
			(5)
			Total 12

7(a)	$b^2 = a^2\left(e^2-1\right) \Rightarrow e^2 = \frac{25}{a^2}+1 = \frac{25+a^2}{a^2}$ oe	**B1**
		(1)
(b)	$x = (\pm)\frac{a}{e} \qquad \frac{x}{a} = (\pm)\frac{y}{5}$	**B1**
	$\frac{a}{e}\times\frac{1}{a} = \pm\frac{y}{5} \Rightarrow y = \pm\frac{5}{e} \Rightarrow AA' = 2\times\frac{5}{e}$ or $\frac{5}{e}-\left(-\frac{5}{e}\right)$	**M1**
	$= \frac{10}{e}$	**A1**
		(3)
(c)	$\frac{1}{2}\times\frac{"10"}{e}\times\left(ae+\frac{a}{e}\right)$ or e.g. $\frac{1}{2}\times\frac{10a}{\sqrt{25+a^2}}\times\left(\sqrt{25+a^2}+\frac{a^2}{\sqrt{25+a^2}}\right)$	**M1**
	$\frac{1}{2}\frac{10}{e}\left(ae+\frac{a}{e}\right) = \frac{164}{3} \Rightarrow 15\left(a+\frac{a}{e^2}\right) = 164$ or $\frac{1}{2}\times\frac{10a}{\sqrt{25+a^2}}\times\left(\sqrt{25+a^2}+\frac{a^2}{\sqrt{25+a^2}}\right) = \frac{164}{3}$	**M1**
	$\Rightarrow 15a\left(1+\frac{a^2}{25+a^2}\right) = 164$	**A1 (M1 on EPEN)**
	$\Rightarrow 15a\left(\frac{25+2a^2}{25+a^2}\right) = 164 \Rightarrow 375a+30a^3 = 164\left(25+a^2\right)$ $\Rightarrow 30a^3-164a^2+375a-4100 = 0*$	**A1***
		(4)
(d)	$30a^3-164a^2+375a-4100 = (3a-20)\left(10a^2+12a+205\right)$	**B1 (M1 on EPEN)**
	$12^2-4(10)(205) = ...$ $10a^2+12a+205 = 10\left(\left(a+\frac{12}{20}\right)^2-\frac{144}{400}\right)+205$	**M1**
	E.g. $12^2-4(10)(205)<0$ so there are no other roots of the equation. Hence $a = \frac{20}{3}$ is only possible value.	**A1**
		(3)
		(11 marks)

Question Number	Scheme	Notes	Marks
3(a)	$y = \text{arsinh}\left(\sqrt{x^2-1}\right)$		
	For all Ways allow the final answer to be written as $\dfrac{1}{\left(x^2-1\right)^{\frac{1}{2}}}$ or $\left(x^2-1\right)^{-\frac{1}{2}}$		
Way 1	$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1+\left(\sqrt{x^2-1}\right)^2}} \times \dfrac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}(2x)$ M1: Obtains $\dfrac{1}{\sqrt{1+\left(\sqrt{x^2-1}\right)^2}} \times f(x)$ or e.g., $\dfrac{1}{x} \times f(x)$ $\quad f(x) \neq k$ A1: Fully correct unsimplified expression		**M1** **A1**
	$= \dfrac{1}{\sqrt{1+x^2-1}} \times \dfrac{x}{\sqrt{x^2-1}} = \dfrac{1}{\sqrt{x^2-1}}$ * or e.g., $= \dfrac{1}{x} \times \dfrac{x}{\sqrt{x^2-1}} = \dfrac{1}{\sqrt{x^2-1}}$ *	Correct completion with intermediate line of working and no errors	**A1***
			(3)
Way 2 **Takes sinh of both sides**	$y = \text{arsinh}\left(\sqrt{x^2-1}\right) \Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \cosh y \dfrac{dy}{dx} = \dfrac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}(2x)$ M1: Takes sinh of both sides and differentiates to obtain $\cosh y \dfrac{dy}{dx} = f(x)$ $\quad f(x) \neq k$ A1: Fully correct unsimplified equation		**M1** **A1**
	$\cosh y = \sqrt{1+\sinh^2 y}$ or $\sqrt{1+x^2-1} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2-1}}$ *	Correct completion with clear use of identity (must see more than just $\cosh y = x$) and no errors	**A1***
			(3)
Way 3 **Takes sinh & squares**	$y = \text{arsinh}\left(\sqrt{x^2-1}\right) \Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \sinh^2 y = x^2-1 \Rightarrow 2\sinh y \cosh y \dfrac{dy}{dx} = 2x$ M1: Takes sinh of both sides, squares and differentiates to obtain $c\sinh y \cosh y \dfrac{dy}{dx} = f(x)$ $\quad f(x) \neq k$ A1: Fully correct unsimplified expression or equation		**M1** **A1**
	$\cosh y = \sqrt{1+\sinh^2 y}$ or $\sqrt{1+x^2-1} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2-1}}$	Correct completion with clear use of identity (must see more than just $\cosh y = x$) and no errors	**A1***
			(3)
Way 4 **Takes sinh & squares & uses identity**	$\Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \sinh^2 y = x^2-1 \Rightarrow \cosh^2 y = 1+\left(x^2-1\right) \Rightarrow \cosh^2 y = x^2 \Rightarrow 2\sinh y \cosh y \dfrac{dy}{dx} = 2x$ M1: Takes sinh of both sides, squares, uses identity and differentiates to obtain $c\sinh y \cosh y \dfrac{dy}{dx} = f(x)$ $\quad f(x) \neq k$ Allow sign errors with identity for the M mark. A1: Fully correct unsimplified expression or equation		**M1** **A1**
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2-1}}$	Correct completion with clear use of identity and no errors	**A1***
			(3)

Question Number	Scheme	Notes	Marks
3(a) Way 5 **Takes sinh & squares & uses identity & roots**	$\Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \sinh^2 y = x^2-1 \Rightarrow \cosh^2 y = 1+(x^2-1) \Rightarrow \cosh y = x \Rightarrow \sinh y\frac{dy}{dx} = 1$ M1: Takes sinh of both sides, squares, uses identity, roots and differentiates to obtain $c\sinh y\frac{dy}{dx} = f(x)$ or k Allow sign errors with identity. A1: Fully correct unsimplified expression or equation		**M1** **A1**
	$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}}$	Correct completion with clear use of identity and no errors	**A1***
			(3)
Way 6 **Uses log form of arsinh first**	$y = \text{arsinh}\left(\sqrt{x^2-1}\right) \Rightarrow y = \ln\left(\sqrt{x^2-1}+\sqrt{x^2-1+1}\right) = \ln\left(\sqrt{x^2-1}+x\right) \Rightarrow \frac{dy}{dx} = \frac{\frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)+1}{\sqrt{x^2-1}+x}$ M1: Use log form of arsinh correctly and differentiates to obtain $\frac{f(x)\neq k}{\sqrt{x^2-1}+x}$ A1: Fully correct unsimplified expression		**M1** **A1**
	$= \frac{\frac{x}{\sqrt{x^2-1}}+1}{\sqrt{x^2-1}+x}$ or $\frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}} \times \frac{1}{\sqrt{x^2-1}+x} = \frac{1}{\sqrt{x^2-1}}$ *	Correct completion with intermediate line of working and no errors	**A1***
			(3)
You may see other variations e.g., using exponential definitions, attempts via dx/dy. The M mark is for differentiating to obtain correct forms and the first A is awarded if it is correct. The final A is for correct completion.			

Question Number	Scheme	Notes	Marks
3(b)	$f(x) = \frac{1}{3}\text{arsinh}\left(\sqrt{x^2-1}\right) - \arctan x$		
	$f'(x) = \frac{1}{3\sqrt{x^2-1}} - \frac{1}{1+x^2}$	$f'(x) = \frac{A}{\sqrt{x^2-1}} \pm \frac{1}{1\pm x^2}$ $A = \frac{1}{3}$, 3 or 1	**M1 (B1 on ePen)**
	$1+x^2 = 3\sqrt{x^2-1}$ $1+2x^2+x^4 = 9x^2-9$	Sets $\frac{A}{\sqrt{x^2-1}} \pm \frac{1}{1+x^2} = 0$ $A = \frac{1}{3}$, 3 or 1 **Denominator of derivative of arctan x must now be $1+x^2$** Cross multiplies and squares to obtain the correct form for both sides so do not condone e.g., $(1+x^2)^2 = 1+x^4$ May see the quartic obtained through equivalent work.	**M1**
	$x^4-7x^2+10=0 \Rightarrow (x^2-2)(x^2-5)=0 \Rightarrow x^2 = 2, 5$ Solves a 3TQ in x^2 (usual rules and one correct root if no working). No requirement to see the terms collected. Ignore labelling of solutions so allow e.g., "$x = 2, 5$". One correct value for their equation if no working, which may be for x or x^2, so just look for the values. May change the variable. Allow for a correct solution with no working from solving a three term quartic of the correct form on a calculator. Allow if value for x^2 is negative or if roots are complex. **Requires previous M marks.**		**ddM1**
	$x = \sqrt{2},\ \sqrt{5}$	Both exact and no other solutions e.g., $\pm$ is A0 unless negatives rejected. Must not reject either correct solution.	**A1**
			(4)
			Total 7

Question	Scheme		Marks
5(a)	$I_n = \left[\cos^{n-1}\theta\sin\theta\right]_0^{\frac{\pi}{4}} - (-)\int_0^{\frac{\pi}{4}}(n-1)\cos^{n-2}\theta\sin^2\theta\,d\theta$		M1A1
	M1: Attempt parts the correct way round A1: Correct expression		
	so $I_n = \left(\frac{1}{\sqrt{2}}\right)^n +$	Uses limits to obtain $\left(\frac{1}{\sqrt{2}}\right)^n$	B1
	i.e. $I_n = \ldots\ldots + \int_0^{\frac{\pi}{4}}(n-1)\cos^{n-2}\theta(1-\cos^2\theta)d\theta$		**d**M1
	M1: Replaces $\sin^2\theta$ by $1 - \cos^2\theta$ **Dependent on the previous method mark**		
	So $I_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2} - (n-1)I_n$, and $nI_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2}$ *		**dd**M1A1cso
	M1: Replaces expressions for I_n and I_{n-1} **Dependent on both previous method marks** A1: Achieves printed answer with no errors seen		
			(6)
	Alternative		
	$I_n = \int_0^{\frac{\pi}{4}} \cos^{n-2}\theta\cos^2\theta\,d\theta = \int_0^{\frac{\pi}{4}} \cos^{n-2}\theta(1-\sin^2\theta)d\theta$		2nd M1
	Writes $\cos^n\theta$ as $\cos^{n-2}\theta\cos^2\theta$ and replaces $\cos^2\theta$ by $1 - \sin^2\theta$		
	$I_n = I_{n-2} + \left[\frac{1}{n-1}\cos^{n-1}\theta\sin\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\frac{1}{(n-1)}\cos^n\theta\,d\theta$		dM1A1
	dM1: Attempt parts the correct way round A1: Correct expression		
	$I_n = I_{n-2} + \frac{1}{n-1}\left(\frac{1}{\sqrt{2}}\right)^n - \frac{1}{n-1}I_n$	B1: Uses limits to obtain $\frac{1}{n-1}\left(\frac{1}{\sqrt{2}}\right)^n$ **dd**M1: Replaces expressions for I_n and I_{n-1}	B1**dd**M1
	$nI_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2}$	Achieves printed answer with no errors seen	A1
(b)	$I_1 = \int_0^{\frac{\pi}{4}}\cos\theta\,d\theta = [\sin\theta]_0^{\frac{\pi}{4}} = \frac{1}{\sqrt{2}}$	M1: Attempt I_1 A1: $\frac{1}{\sqrt{2}}$	M1A1
	$I_3 = \frac{1}{3}(\frac{1}{2\sqrt{2}} + 2I_1)$, $I_5 = \frac{1}{5}(\frac{1}{4\sqrt{2}} + 4I_3)$ or $3I_3 = \frac{1}{2\sqrt{2}} + 2I_1$, $5I_5 = \frac{1}{4\sqrt{2}} + 4I_3$	M1: Uses reduction formula first time (allow slips providing the reduction formula is being used) M1: Uses reduction formula second time (allow slips providing the reduction formula is being used)	M1M1
	$I_5 = \frac{43\sqrt{2}}{120}$ or $\frac{43}{60\sqrt{2}}$		A1

4	$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$		
(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{b\sec^2\theta}{a\sec\theta\tan\theta}$ or $\dfrac{b^2x}{a^2y}$ or $\dfrac{bx}{a^2}\left(\dfrac{x^2}{a^2}-1\right)^{-\frac{1}{2}}$	Correct tangent gradient in any form	B1
	$m_N=-\dfrac{a\sec\theta\tan\theta}{b\sec^2\theta}\left(=-\dfrac{a}{b}\sin\theta\right)$	Use parametric forms and the correct perpendicular rule	M1
	$y-b\tan\theta=-\dfrac{a}{b}\sin\theta(x-a\sec\theta)$	M1: Correct straight line method using their m_N Use of $y=mx+c$ must include finding a value for c A1: Correct equation any equivalent to that shown.	M1A1
	$by-b^2\tan\theta=-ax\sin\theta+a^2\tan\theta$		
	$ax\sin\theta+by=\left(a^2+b^2\right)\tan\theta$ *	Completes to printed answer with at least one intermediate step	A1*
			(5)
(b)	$y=0\Rightarrow x=\dfrac{\left(a^2+b^2\right)\tan\theta}{a\sin\theta}\left(=\dfrac{\left(a^2+b^2\right)}{a}\sec\theta\right)$	Correct x coordinate	B1
	M is $\left(\dfrac{1}{2}\left(\dfrac{a^2+b^2}{a}\sec\theta+a\sec\theta\right),\dfrac{b}{2}\tan\theta\right)$ $=\left(\dfrac{2a^2+b^2}{2a}\sec\theta,\dfrac{b}{2}\tan\theta\right)$ oe	M1: Correct midpoint method for their x coordinate A1: Correct coordinates for M, any equivalent accepted. Need not be in coordinate brackets.	M1A1
			(3)
(c)	$\sec\theta=\dfrac{2ax}{2a^2+b^2},\tan\theta=\dfrac{2y}{b}\Rightarrow 1+\left(\dfrac{2y}{b}\right)^2=\left(\dfrac{2ax}{2a^2+b^2}\right)^2$	M1: Correct attempt to eliminate θ using coordinates of M A1: Correct equation	M1A1
	$y^2=\dfrac{b^2}{4}\left(\dfrac{4a^2x^2}{\left(2a^2+b^2\right)^2}-1\right)$ oe	dM1: Makes y^2 the subject A1: Correct equation in the required form	dM1A1
			(4)
			Total 12

Question Number	Scheme	Notes	Marks
3(a)	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech } y = \frac{x}{2}$ $\Rightarrow \frac{dx}{dy} = -2\text{sech } y \tanh y$	Takes "sech" of both sides and differentiates to obtain $\frac{dx}{dy} = k \text{ sech } y \tanh y$ or equivalent.	M1
	$\Rightarrow \frac{dx}{dy} = -2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2}$ M1: Replaces sech y with $\frac{x}{2}$ and tanh y with $\sqrt{1-\left(\frac{x}{2}\right)^2}$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
			(4)
(a) Way 2	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech } y = \frac{x}{2}$ $\Rightarrow \cosh y = \frac{2}{x} \Rightarrow \sinh y \frac{dy}{dx} = -\frac{2}{x^2}$	Takes "sech" of both sides, changes to "cosh" and differentiates to obtain $\sinh y \frac{dy}{dx} = \frac{k}{x^2}$ or equivalent.	M1
	$\Rightarrow \frac{dy}{dx} = -\frac{2}{x^2 \sinh y} = -\frac{2}{x^2\sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ M1: Replaces sinh y with $\sqrt{\left(\frac{2}{x}\right)^2 - 1}$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
(a) Way 3	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow y = \text{arcosh}\left(\frac{2}{x}\right)$ Changes to "arcosh" correctly. **Score this as the second M mark on EPEN.**		M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{2}{x}\right)^2 - 1}} \times -\frac{2}{x^2}$ M1: Differentiates to the form $\frac{k}{x^2\sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ oe A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1

(b)	$f(x)=\tanh^{-1}(x)+\text{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow f'(x)=\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}$ Correct f '(x) following through their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ Also allow with "made up" p and q or the letters p and q.		B1ft
	$\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2(1-x^2)=x\sqrt{4-x^2}\Rightarrow 4(1-x^2)^2=x^2(4-x^2)$ Sets $\frac{dy}{dx}=0$ with their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ and squares both sides to reach a quartic equation		M1
	$5x^4-12x^2+4=0$	Correct quartic	A1
	$5x^4-12x^2+4=0\Rightarrow x^2=2,\ 0.4$ $\Rightarrow x=\ldots$	Solves their quartic equation to obtain a value for x^2 and proceeds to a value for x. Apply usual rules for solving and check if necessary. Allow complex roots.	M1
	$x=\sqrt{\frac{2}{5}}$	Correct exact answer (allow equivalents e.g. $\frac{\sqrt{10}}{5}$). If any extra answers given score A0 e.g. $x=\pm\sqrt{\frac{2}{5}}$	A1
			(5)
			Total 9

3(a) Way 1 Identities first then squares	$y=\frac{1}{2}(\tan x+\cot x)\Rightarrow\frac{dy}{dx}=\frac{1}{2}(\sec^2 x-\text{cosec}^2 x)$ oe	Correct derivative. Any equivalent.	B1
	$=\frac{1}{2}\left(1+\tan^2 x-(1+\cot^2 x)\right)\quad\left\{=\frac{1}{2}(\tan^2 x-\cot^2 x)\right\}$	Applies $\sec^2 x=\pm\tan^2 x\pm 1$ and $\text{cosec}^2 x=\pm\cot^2 x\pm 1$ to their derivative	M1
	$\left(\frac{dy}{dx}\right)^2=\frac{1}{4}(\tan^4 x+\cot^4 x-2\tan^2 x\cot^2 x)$	Squares to a 3 term expression (or 4 if middle terms uncollected) $2\tan^2 x\cot^2 x$ can be seen as 2 **Requires previous M mark.**	**d**M1
	$\left\{1+\left(\frac{dy}{dx}\right)^2=1+\frac{1}{4}(\tan^4 x+\cot^4 x-2)\right\}$ $\Rightarrow\frac{1}{4}(\tan^4 x+\cot^4 x+2)$ or $\frac{1}{4}\tan^4 x+\frac{1}{4}\cot^4 x+\frac{1}{2}$ **Not implied. Must be seen**	Adds the 1 and achieves either expression shown but allow the constant to be multiplied by $\tan^2 x\cot^2 x$ May be seen as e.g., $\frac{1}{2}\sqrt{\tan^4 x+\cot^4 x+2\tan^2 x\cot^2 x}$	A1
	$s=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(\tan^2 x+\cot^2 x)\,dx$ * Allow $\int\frac{1}{2}(\tan^2 x+\cot^2 x)$ or $\frac{1}{2}\int\tan^2 x+\cot^2 x$	M1: Applies the arc length formula with their $\frac{dy}{dx}$ A1: Correct result achieved with no clear mathematical errors seen. Condone omission of "dx" and/or limits and **occasional** missing arguments.	M1 A1*
	Converting to sin & cos: likely to score max of 100010 unless tan & cot are convincingly recovered		**(6)**

3(b)	$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\tan^2 x+\cot^2 x\right)\,dx=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sec^2 x-1+\text{cosec}^2 x-1\right)\,dx$	Applies $\tan^2 x=\pm\sec^2 x\pm1$ and $\cot^2 x=\pm\text{cosec}^2 x\pm1$ to the integral	M1
	Work in sin and cos must use identities (sign errors only) and lead to a result of the form below after integration condoning the absence of a term in x but allow the last M to be available following a completed attempt at integration.		
	$=\frac{1}{2}\left[\tan x-\cot x-2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1: For $\pm\sec^2 x\to\pm\tan x$ and $\pm\text{cosec}^2 x\to\pm\cot x$ **Requires previous M mark.** A1: Correct integration. Limits not required.	dM1 A1
	$\frac{1}{2}\left(\tan\frac{\pi}{3}-\cot\frac{\pi}{3}-\frac{2\pi}{3}-\left(\tan\frac{\pi}{6}-\cot\frac{\pi}{6}-\frac{2\pi}{6}\right)\right)$ $\left\{\frac{1}{2}\left(\sqrt{3}-\frac{2\pi}{3}-\frac{\sqrt{3}}{3}-\left(\frac{\sqrt{3}}{3}-\frac{\pi}{3}-\sqrt{3}\right)\right)\right\}$	Applies the limits (see note below) following any completed attempt at integration. Allow slips provided it is a clear attempt at $f\left(\frac{\pi}{3}\right)-f\left(\frac{\pi}{6}\right)$	M1
	Correct answer in any exact simplified form with 2 terms e.g. $\frac{1}{2}\left(\frac{4\sqrt{3}}{3}-\frac{\pi}{3}\right)$, $\frac{2\sqrt{3}}{3}-\frac{\pi}{6}$, $\frac{2}{\sqrt{3}}-\frac{\pi}{6}$, $\frac{1}{3}\left(2\sqrt{3}-\frac{\pi}{2}\right)$, $\frac{4\sqrt{3}-\pi}{6}$		A1
	Note they may apply the limits $\frac{\pi}{4}$ & $\frac{\pi}{6}$ or $\frac{\pi}{3}$ & $\frac{\pi}{4}$ and then double the result.		**(5)**
	Just the answer or decimal answer (0.6311017628) is 0/5		**Total 11**