Question Number	Scheme	Notes	Marks
2(a) **Way 1** **TU = I**	$\mathbf{TU}=\mathbf{I}\Rightarrow\begin{pmatrix}2&3&7\\3&2&6\\a&4&b\end{pmatrix}\begin{pmatrix}6&-1&-4\\15&c&-9\\-8&a&5\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $\Rightarrow$ e.g., $\begin{matrix}6a+60-8b=0\\-4a-36+5b=1\end{matrix}$ or $\begin{matrix}-2+3c+7a=0\\-3+2c+6a=1\end{matrix}$ Obtains at least 2 equations with at least one correct. (condone column × row multiplication leading to the way 2 equations – see below). Ignore errors in unused elements or equations.		M1
	e.g., $\begin{matrix}6a-8b=-60\\-4a+5b=37\end{matrix}\Rightarrow a=\ldots,\ b=\ldots$ **or** $\begin{matrix}7a+3c=2\\6a+2c=4\end{matrix}\Rightarrow a=\ldots,\ c=\ldots$ Obtains values for two of a, b and c. You do **not** need to check their values. As long as the previous M mark was scored, it is sufficient to just write down values.		dM1
	$a=2,\ b=9,\ c=-4$	A1: Two correct values A1: All three correct values and no extra values unless they are rejected.	A1 A1
			(4)
Way 2 **UT = I** **For first 2 marks**	$\mathbf{UT}=\mathbf{I}\Rightarrow\begin{pmatrix}6&-1&-4\\15&c&-9\\-8&a&5\end{pmatrix}\begin{pmatrix}2&3&7\\3&2&6\\a&4&b\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $12-3-4a=1$ $\Rightarrow$ e.g., $42-6-4b=0$ $[45+2c-36=1]$ Obtains at least 2 equations with at least one correct. (condone column × row multiplication leading to the way 1 equations – see above). Ignore errors in unused elements or equations.		M1
	e.g., $-4a=-8,\ -4b=-36\ [2c=-8]$ $\Rightarrow a=\ldots,\ b=\ldots$ Obtains values for two of a, b and c. You do **not** need to check their values. As long as the previous M mark was scored, it is sufficient to just write down values.		dM1
Way 3 **Inverses** **For first mark**	$\mathbf{T}^{-1}=\mathbf{U}\Rightarrow\frac{1}{4a-5b+36}\begin{pmatrix}2b-24&-3b+28&4\\6a-3b&-7a+2b&9\\-2a+12&3a-8&-5\end{pmatrix}=\begin{pmatrix}6&-1&-4\\15&c&-9\\-8&a&5\end{pmatrix}$ $\Rightarrow$ e.g., $\frac{4}{4a-5b+36}=-4,\ \frac{2b-24}{4a-5b+36}=6\ \left[\frac{-7a+2b}{4a-5b+36}=c\right]$ For $\mathbf{T}^{-1}=\frac{1}{\mathrm{f}(a,b)}\mathbf{M}$ where $\mathbf{M}$ has at least 1 correct element **and** obtains 2 equations. Note that there is no requirement to find all the elements of $\mathbf{M}$. **OR** $\mathbf{U}^{-1}=\mathbf{T}\Rightarrow\frac{1}{-6a-2c+3}\begin{pmatrix}9a+5c&-4a+5&4c+9\\-3&-2&-6\\15a+8c&-6a+8&6c+15\end{pmatrix}=\begin{pmatrix}2&3&7\\3&2&6\\a&4&b\end{pmatrix}$ $\Rightarrow$ e.g., $\frac{-3}{-6a-2c+3}=3,\ \frac{4c+9}{-6a-2c+3}=7\ \left[\frac{6c+15}{-6a-2c+3}=b\right]$ For $\mathbf{U}^{-1}=\frac{1}{\mathrm{f}(a,c)}\mathbf{M}$ where $\mathbf{M}$ has at least 1 correct element **and** obtains 2 equations Note that there is no requirement to find all the elements of $\mathbf{M}$.		M1

Question	Scheme	Notes	Marks
2(b)	$\frac{x-1}{3}=\frac{y}{-4}=z+2 \Rightarrow [l_2 : \mathbf{r} =] \begin{pmatrix}1\\0\\-2\end{pmatrix} \pm \lambda \begin{pmatrix}3\\-4\\1\end{pmatrix}$ or $\left(\mathbf{r}-\begin{pmatrix}1\\0\\-2\end{pmatrix}\right)\times\begin{pmatrix}3\\-4\\1\end{pmatrix}=\mathbf{0}$ Obtains parametric/vector form (allow one slip only) or clearly identifies position and direction vectors. May be implied by an attempt to transform both.		M1
	$\begin{pmatrix}6 & -1 & -4\\15 & '-4' & -9\\-8 & '2' & 5\end{pmatrix}\begin{pmatrix}1+3\lambda\\-4\lambda\\-2+\lambda\end{pmatrix}=\begin{pmatrix}6+18\lambda+4\lambda+8-4\lambda\\15+45\lambda+16\lambda+18-9\lambda\\-8-24\lambda-8\lambda-10+5\lambda\end{pmatrix}$ **or** their **U** $\times$ their $\begin{pmatrix}1 & 3\\0 & -4\\-2 & 1\end{pmatrix}$ **or** $\times$their $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ **and** $\times$their $\begin{pmatrix}3\\-4\\1\end{pmatrix}$ **or** their **U** $\times$their $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ **and U** $\times$e.g. $\begin{pmatrix}4\\-4\\-1\end{pmatrix}$ then $dir=\begin{pmatrix}32\\85\\-45\end{pmatrix}-\begin{pmatrix}14\\33\\-18\end{pmatrix}$ Complete and correct method with their b and c for their **U** × their parametric form or **U** × both vectors or **U** × 2 points on the line and attempts direction. Must be an attempt to mutliply correctly i.e. clearly not row×row but allow attempts that use $\mathbf{T}^{-1}$ for **U** using their a and b provided all elements are constants and it is a "changed" **T** **OR** $\begin{pmatrix}2 & 3 & 7\\3 & 2 & 6\\"2" & 4 & "9"\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+3\lambda\\-4\lambda\\-2+\lambda\end{pmatrix}\Rightarrow \begin{matrix}2x+3y+7z=1+3\lambda\\3x+2y+6z=-4\lambda\\2x+4y+9z=-2+\lambda\end{matrix}$ $x=18\lambda+14$ $\Rightarrow y=52\lambda+33$ $z=-18-27\lambda$ A complete method using their parametric form and their **T** to produce and solve 3 simultaneous equations to find x, y and z in terms of λ Alternatively solves $\mathbf{T}x=("\mathbf{i}-2\mathbf{k}")$ and $\mathbf{T}x=("3\mathbf{i}-4\mathbf{k}+\mathbf{k}")$ to find position and direction		M1
	$[l_1 : \mathbf{r} =]\begin{pmatrix}14+18\lambda\\33+52\lambda\\-18-27\lambda\end{pmatrix}$ $\Rightarrow \frac{x-14}{18}=\frac{y-33}{52}=\frac{z+18}{-27}$	dM1: Correctly converts their result into Cartesian equation. **Requires previous method mark** A1: Correct Cartesian equation - allow equivalents e.g., $\ldots=\frac{z-(-18)}{-27}$, $\ldots=\frac{-z-18}{27}$	dM1 A1
			(4)
			Total 8

2(b) Alternative

$$x = t \Rightarrow y = \frac{4}{3} - \frac{4}{3}t,\ \ z = \frac{1}{3}t - \frac{7}{3}$$

M1: Obtains parametric form (allow one slip only)

$$\begin{pmatrix} 6 & -1 & -4 \\ 15 & '-4' & -9 \\ -8 & '2' & 5 \end{pmatrix}\begin{pmatrix} t \\ \frac{4}{3} - \frac{4}{3}t \\ \frac{1}{3}t - \frac{7}{3} \end{pmatrix} = \begin{pmatrix} 6t - \frac{4}{3} + \frac{4}{3}t - \frac{4}{3}t + \frac{28}{3} \\ 15t - \frac{16}{3} + \frac{16}{3}t - 3t + 21 \\ -8t + \frac{8}{3} - \frac{8}{3}t + \frac{5}{3}t - \frac{35}{5} \end{pmatrix}$$

M1: As above

$$[l_1 : \mathbf{r} =]\begin{pmatrix} 8 + 6t \\ \frac{47}{3} + \frac{52}{3}t \\ -9 - 9t \end{pmatrix}$$

$$\Rightarrow \frac{x-8}{6} = \frac{y - \frac{47}{3}}{\frac{52}{3}} = \frac{z+9}{-9}$$

dM1A1: As above

Question Number	Scheme	Notes	Marks
8(a)	$\mathbf{n}=\begin{pmatrix}1\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}-10+6\\-(2-9)\\-2+15\end{pmatrix}$	Attempt vector product between normal vectors	M1
	$=\begin{pmatrix}-4\\7\\13\end{pmatrix}$	Correct vector	A1
	$x=0\Rightarrow -5y+3z=11,\quad -2y+2z=7$ $\Rightarrow y=-\frac{1}{4}, z=\frac{13}{4}$ or $y=0\Rightarrow x+3z=11,\quad 3x+2z=7$ $\Rightarrow x=-\frac{1}{7}, z=\frac{26}{7}$ or $z=0\Rightarrow x-5y=11, 3x-2y=7$ $\Rightarrow x=1, y=-2$	Correct strategy to find a point on l	M1
		Correct position vector of point on l	A1
	$\mathbf{r}=\mathbf{i}-2\mathbf{j}+\lambda(-4\mathbf{i}+7\mathbf{j}+13\mathbf{k})$	Correct equation. (follow through their position and direction vectors but must be "$\mathbf{r}$ =")	A1ft
			(5)
ALT	$x=11+5y-3z$		
	$3x-2y+2z=7\Rightarrow 3(11+5y-3z)-2y+2z=7$ $\Rightarrow y-\frac{7z}{13}=-\frac{26}{13}\quad\left(z=\frac{13y+26}{7}\right)$ Eliminate one variable		M1
	$x=11+5\left(-\frac{26}{13}+\frac{7z}{13}\right)\Rightarrow z=\frac{13-13x}{4}$	Obtain 2 correct expressions for one of the variables	A1
	$\dfrac{x-1}{-\frac{4}{13}}=\dfrac{y+2}{\frac{7}{13}}=z$	M1 Obtain a Cartesian equation for l A1 Correct equation	M1A1
	$\mathbf{r}=(\mathbf{i}-2\mathbf{j})+\lambda\left(-\frac{4}{13}\mathbf{i}+\frac{7}{13}\mathbf{j}+\mathbf{k}\right)$ oe	Deduce a vector equation for l Follow through their Cartesian equation	A1ft
			(5)

(b)	$\begin{pmatrix}3\\2\\1\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$	Correct vector joining P to Q	B1
	$\begin{pmatrix}-4\\7\\13\end{pmatrix}\times\begin{pmatrix}1\\2\\-2\end{pmatrix}=\begin{pmatrix}-40\\5\\-15\end{pmatrix}$	Attempt vector product between the direction of l and their $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$	M1
		Correct vector	A1
	$\sin\theta=\frac{\lvert -40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert -4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert\lvert \mathbf{i}+2\mathbf{j}-2\mathbf{k}\rvert}$	Angle between PQ and line n	
	$d=\lvert\overrightarrow{PQ}\rvert\sin\theta$		
	$d=\frac{\lvert -40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert -4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert}=\frac{1}{\sqrt{234}}\sqrt{40^2+5^2+15^2}$	Fully correct method for the distance	M1
	$d=\frac{5\sqrt{481}}{39}$	Cao Allow equivalent **exact** forms e.g. $d=\frac{5\sqrt{74}}{\sqrt{234}}$	A1
			(5)
ALT 1	$\mathbf{r}_m=\begin{pmatrix}2\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ or $\mathbf{r}_n=\begin{pmatrix}3\\2\\1\end{pmatrix}+\mu\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$	Vector equation for either line with their direction vector from (a)	B1ft
	$\overrightarrow{OP}=\begin{pmatrix}2\\0\\3\end{pmatrix}$ $\overrightarrow{ON}=\begin{pmatrix}3-\frac{4}{7}\mu\\2+\mu\\1+\frac{13}{7}\mu\end{pmatrix}$ $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}$	Uses either P and the parametric form of a point on n OR Q and the parametric form of a point on m	
	$\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}.\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}=0$	M1: Forms scalar product of vector NP and direction vector of l and equates to zero A1: Correct vectors	M1A1
	$\Rightarrow\mu=\frac{56}{117}$	Solves	M1
	$\Rightarrow d=\sqrt{\left(-\frac{85}{117}\right)^2+\left(-\frac{290}{117}\right)^2+\left(\frac{10}{9}\right)^2}=\frac{5\sqrt{481}}{39}$	Obtains the correct distance	A1
			(5)

Question	Scheme	Marks
6(a)	$\begin{pmatrix}2\\0\\7\end{pmatrix}\times\begin{pmatrix}3\\-2\\3\end{pmatrix}=\begin{pmatrix}0-(-14)\\-(6-21)\\-4-0\end{pmatrix};=\begin{pmatrix}14\\15\\-4\end{pmatrix}$	M1; A1
		(2)
(b)	Direction of l is $\pm\left(\begin{pmatrix}1\\7\\3\end{pmatrix}-\begin{pmatrix}-1\\5\\2\end{pmatrix}\right)=\pm\begin{pmatrix}2\\2\\1\end{pmatrix}$	B1
	Angle between line and normal to plane is given by $\left(\cos\beta=\right)\frac{14\times2+15\times2-4\times1}{\sqrt{437}\sqrt{9}}\left(=\frac{18}{\sqrt{437}}\right)$	M1
	So angle between plane and line is $\alpha=90°-\beta=...$	M1
	$=59°$	A1
		(4)
(c)	$\left\{\text{Equation of plane is } \mathbf{r}.\begin{pmatrix}14\\15\\-4\end{pmatrix}=\right\}\begin{pmatrix}-1\\5\\2\end{pmatrix}.\begin{pmatrix}14\\15\\-4\end{pmatrix}=53$	M1 A1
	So perpendicular distance is $\frac{\lvert14\times1+15\times7-4\times3-53\rvert}{\sqrt{437}}$	M1
	$=\frac{54}{\sqrt{437}}$	A1
		(4)
		(10 marks)

Question Number	Scheme	Notes	Marks
9	May use **i**, **j**, **k** notation		
9(a)	$\mathbf{n}=\begin{pmatrix}3\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\-2\\2\end{pmatrix}=\ldots \quad \left\{\begin{pmatrix}2\\-5\\-6\end{pmatrix}\right\}$	Calculates the vector product of two vectors in Π_1 (two components correct)	**M1**
	$\begin{pmatrix}5\\3\\0\end{pmatrix}.\begin{pmatrix}2\\-5\\-6\end{pmatrix}=\ldots \quad \{-5\}$	Calculates the scalar product of a point in the plane and their normal. Not dependent but must follow an attempt at a vector product which could be poor, e.g., $3\mathbf{i}+2\mathbf{k}$. Value must be correct if there is no indication of a correct method to evaluate the scalar product.	**M1**
	$\mathbf{r}.\begin{pmatrix}2\\-5\\-6\end{pmatrix}=\begin{pmatrix}5\\3\\0\end{pmatrix}.\begin{pmatrix}2\\-5\\-6\end{pmatrix}\Rightarrow 2x-5y-6z=-5$	Any correct Cartesian equation, e.g., $-2x+5y+6z=5$ $2x-5y-6z+5=0$	**A1**
			(3)
Alt **Sim eqns**	$x=5+3s+t$ $y=3-2t \quad \Rightarrow$ e.g., $y+z=3+s$ $z=s+2t$	Forms simultaneous equations in x, y, z, s and t and obtains an equation that eliminates at least one of s and t	**M1**
	$x=5+3(y+z-3)+\frac{1}{2}z-\frac{1}{2}(y+z-3)$ $x=\frac{5}{2}y+3z-\frac{5}{2}$	M1: Proceeds to an equation in x, y and z only A1: Any correct equation with terms collected	**M1** **A1**
			(3)

Question Number	Scheme	Notes	Marks
9(b) **Way 1**	$2x-5y-6z=-5,\quad 5x-2y+3z=1$ $\Rightarrow$ e.g., $12x-9y=-3$	Uses both plane equations to eliminate one variable. May see $21y+36z=27,\ 21x+27z=15$	**M1**
	e.g., $4x-3y=-1\Rightarrow \underline{\underline{x=\frac{3y-1}{4}}}\Rightarrow \underline{y=\frac{4x+1}{3}}$ $3z=1-\frac{5(3y-1)}{4}+2y=\frac{4-15y+5+8y}{4}\Rightarrow \underline{\underline{z=\frac{9-7y}{12}}}\Rightarrow \underline{y=\frac{12z-9}{-7}}$ Expresses one variable in terms of the other two (single underlining) or expresses two variables in terms of the other one (double underlining). This work may be seen by setting a variable equal to a parameter to find the other variables in terms of the parameter (or the parameter in terms of the other two variables) e.g., $y=\lambda,\ \ x=\mathrm{f}(\lambda),\ \ z=\mathrm{g}(\lambda)\ \ \left\{\Rightarrow x=\frac{-1+3\lambda}{4},\ y=\lambda,\ z=\frac{9-7\lambda}{12}\right\}$ $y=\lambda,\ \ \lambda=\mathrm{f}(x),\ \ \lambda=\mathrm{g}(z)\ \ \left\{\Rightarrow \lambda=\frac{4x+1}{3},\ y=\lambda,\ \lambda=\frac{12z-9}{-7}\right\}$ See examples below. **Requires previous M mark.**		**dM1**
	e.g., $\frac{4x+1}{3}=y=\frac{12z-9}{-7}\Rightarrow\frac{x+\frac{1}{4}}{\frac{3}{4}}=\frac{y-0}{1}=\frac{z-\frac{3}{4}}{-\frac{7}{12}}$ or e.g., $x=\frac{-1+3\lambda}{4},\ \ y=\lambda,\ \ z=\frac{9-7\lambda}{12}\Rightarrow$ $\Rightarrow \mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}\frac{3}{4}\\1\\-\frac{7}{12}\end{pmatrix}$ or e.g. $\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}9\\12\\-7\end{pmatrix}$	ddM1: Correct method to form RHS of vector equation. Allow slips but must not be a clearly incorrect method (e.g., point and direction confused, all non-zero point coordinates the wrong sign, no attempt seen or implied to obtain single coefficients for the variables in the numerator where necessary). Allow this mark if the point is later changed by multiplication e.g., $\left(-\frac{1}{4},\ 0,\ \frac{3}{4}\right)$ becomes $(-1,\ 0,\ 3)$ Condone missing $\mathbf{r}=$... Allow this mark if $(\mathbf{r}-\mathbf{a})\times\mathbf{b}\ (=0)$ or $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ are appropriately used. **Requires both previous M marks.** A1: Any correct **equation** (with any parameter). Do not condone e.g., $l=$ ... Do not isw if the point is changed by multiplication.	**ddM1** **A1**
examples	$x=\frac{3y-1}{4}=\frac{5-9z}{7}\Rightarrow\frac{x-0}{1}=\frac{y-\frac{1}{3}}{\frac{4}{3}}=\frac{z-\frac{5}{9}}{-\frac{7}{9}}$ or $x=\lambda,\ y=\frac{4\lambda+1}{3},\ z=\frac{5-7\lambda}{9}\Rightarrow\mathbf{r}=\begin{pmatrix}0\\\frac{1}{3}\\\frac{5}{9}\end{pmatrix}+\lambda\begin{pmatrix}1\\\frac{4}{3}\\-\frac{7}{9}\end{pmatrix}$ $\frac{5-7x}{9}=\frac{9-7y}{12}=z\Rightarrow\frac{x-\frac{5}{7}}{-\frac{9}{7}}=\frac{y-\frac{9}{7}}{-\frac{12}{7}}=\frac{z-0}{1}$ or $x=\frac{5-9\lambda}{7},\ y=\frac{12z-9}{-7},\ z=\lambda\Rightarrow\mathbf{r}=\begin{pmatrix}\frac{5}{7}\\\frac{9}{7}\\0\end{pmatrix}+\lambda\begin{pmatrix}-\frac{9}{7}\\-\frac{12}{7}\\1\end{pmatrix}$		**(4)**

9(b) **Way 2** **Finds point and takes vector product of normals**	Work may be minimal if they obtain a correct point. But do not accept just sight of an incorrect point without some evidence of an appropriate method to obtain it.		
	$2x-5y-6z=-5,\quad 5x-2y+3z=1$ Let $y=0 \Rightarrow 2x-6z=-5,\quad 5x+3z=1$ or $\Rightarrow$ e.g., $12x-9y=-3$	Assigns a value to one variable to obtain two equations in the other variables or eliminates one variable as in Way 1.	**M1**
	$\Rightarrow 12x=-3 \Rightarrow x=-\frac{1}{4},\ y=0,\ z=\frac{3}{4}$ May see $\left(0,\ \frac{1}{3},\ \frac{5}{9}\right)$ or $\left(\frac{5}{7},\ \frac{9}{7},\ 0\right)$	Solves or assigns a value to one variable to find values for the other variables. There is no need to check a point that arises from no working provided it is clear that the previous M mark has been scored. **Requires previous M mark.**	**dM1**
	Note that a point could be obtained via substituting the given form of Π_1 into Π_2 and expanding (M1) and then finding values of *s* and *t* that satisfy the equation and then finding a point (dM1)		
	$\begin{pmatrix}2\\-5\\-6\end{pmatrix}\times\begin{pmatrix}5\\-2\\3\end{pmatrix}=\begin{pmatrix}-27\\-36\\21\end{pmatrix}\Rightarrow\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}-27\\-36\\21\end{pmatrix}$	Calculates vector product of normals (two components correct) **and** forms RHS of vector equation (allowing for copying slips but must not confuse point and direction). Allow this mark if the point is later changed by multiplication. Condone missing $\mathbf{r}=$... Allow this mark if $(\mathbf{r}-\mathbf{a})\times\mathbf{b}(=0)$ or $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ are appropriately used. **Requires both previous M marks.**	**ddM1**
	$\Rightarrow \mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}-27\\-36\\21\end{pmatrix}$ or e.g., $\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}-9\\-12\\7\end{pmatrix}$	Any correct **equation** in this form (with any parameter). Do not condone e.g., $l=$ ... Do not isw if the point is changed by multiplication. Correct points will have the form $\left(\frac{3\alpha-1}{4},\ \alpha,\ \frac{9-7\alpha}{12}\right)$	**A1**
			(4)
Way 3 **2 points**	Finding 2 points on the line and **subtract** for direction e.g., Finds $\left(-\frac{1}{4},\ 0,\ \frac{3}{4}\right)$ (M1dM1 as Way 2) Then finds $\left(0,\ \frac{1}{3},\ \frac{5}{9}\right)\Rightarrow$ direction $=\left(\frac{1}{4},\ \frac{1}{3},\ -\frac{7}{36}\right)\Rightarrow$ forms RHS of vector equation (ddM1) Then A1 for a correct equation		
			(4)
	Correct points/positions include: $\begin{pmatrix}0\\\frac{1}{3}\\\frac{5}{9}\end{pmatrix}$ $\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}$ $\begin{pmatrix}\frac{5}{7}\\\frac{9}{7}\\0\end{pmatrix}$ $\begin{pmatrix}1\\\frac{5}{3}\\-\frac{2}{9}\end{pmatrix}$ $\begin{pmatrix}\frac{1}{2}\\1\\\frac{1}{6}\end{pmatrix}$ $\begin{pmatrix}-\frac{4}{7}\\-\frac{3}{7}\\1\end{pmatrix}$ $\begin{pmatrix}-1\\-1\\\frac{4}{3}\end{pmatrix}$ $\begin{pmatrix}2\\3\\-1\end{pmatrix}$		

Question Number	Scheme	Notes	Marks
9(c)	Note that use of their line from part (b) must be seen to score any marks in (c)		
	$\mathbf{r}=\begin{pmatrix}-\frac{1}{4}\\0\\\frac{3}{4}\end{pmatrix}+\lambda\begin{pmatrix}9\\12\\-7\end{pmatrix}=\begin{pmatrix}-\frac{1}{4}+9\lambda\\12\lambda\\\frac{3}{4}-7\lambda\end{pmatrix}$ $4\left(-\frac{1}{4}+9\lambda\right)-3(12\lambda)-\left(\frac{3}{4}-7\lambda\right)=0\Rightarrow 7\lambda=\frac{7}{4}\Rightarrow\lambda=\frac{1}{4}$	Substitutes the parametric form of their line (allow slips but must not clearly confuse position and direction) from (b) into Π_3 and solves for λ The "=0" could be implied by a solution.	**M1**
	$\Rightarrow\left(9\left(\frac{1}{4}\right)-\frac{1}{4},\ 12\left(\frac{1}{4}\right),\ -7\left(\frac{1}{4}\right)+\frac{3}{4}\right)=\ldots$ Substitutes their λ into their line and obtains a point/position vector with values for all coordinates/components. If there is no working at least two coordinates/components should be consistent with their equation or parametric form. Isw if the point/position is altered by multiplication. **Requires previous M mark.**		**dM1**
	$(2,\ 3,\ -1)$	Correct point. No others. Allow $x=\ldots,\ y=\ldots,\ z=\ldots$ and condone as a position vector. Do not isw.	**A1**
			(3)
			Total 10
			PAPER TOTAL 75

Question Number	Scheme	Notes	Marks
7(a)	$(3,\ 3,\ -2),\quad \frac{x-1}{2}=\frac{y-2}{-1}=\frac{z+1}{4}$		
	$2\mathbf{i}-\mathbf{j}+4\mathbf{k},\ 2\mathbf{i}+\mathbf{j}-\mathbf{k}$	2 correct vectors lying in Π_1	B1
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&-1&4\\2&1&-1\end{vmatrix}=\begin{pmatrix}-3\\10\\4\end{pmatrix}$	M1: Attempt normal vector using 2 vectors lying in Π_1. If the method is unclear, at least 2 components should be correct. A1: Correct normal (any multiple)	M1A1
	$\begin{pmatrix}-3\\10\\4\end{pmatrix}\bullet\begin{pmatrix}3\\3\\-2\end{pmatrix}=-9+30-8=13$	Attempt scalar product with a point lying in the plane . **Dependent on the previous method mark.**	dM1
	$3x-10y-4z=-13*$	Correct equation	A1*
			(5)

(b)	$\begin{pmatrix}-3\\10\\4\end{pmatrix}\bullet\begin{pmatrix}\alpha\\1\\1\end{pmatrix}=14-3\alpha$ or $\left(\begin{pmatrix}\alpha\\1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\-2\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-10\\-4\end{pmatrix}=3\alpha-1$	Attempt scalar product between $\begin{pmatrix}\alpha\\1\\1\end{pmatrix}$ and their $\begin{pmatrix}-3\\10\\4\end{pmatrix}$ or $\begin{pmatrix}\alpha\\1\\1\end{pmatrix}$ – (a point in the plane Π_1) and their $\begin{pmatrix}-3\\10\\4\end{pmatrix}$	M1
	$\therefore d=\left\lvert\frac{3\alpha-1}{\sqrt{3^2+10^2+4^2}}\right\rvert$ or $\therefore d=\left\lvert\frac{13}{\sqrt{3^2+10^2+4^2}}-\frac{14-3\alpha}{\sqrt{3^2+10^2+4^2}}\right\rvert$	Use of correct distance method. **Dependent on the previous method mark.** (Modulus not needed here)	dM1
	Note: $d=\left\lvert\frac{3\alpha-10-4\pm13}{\sqrt{3^2+10^2+4^2}}\right\rvert$ could score the first 2 method marks		
	$\left\lvert\frac{3\alpha-1}{5\sqrt{5}}\right\rvert=\frac{1}{\sqrt{5}}$	Set their distance = $\frac{1}{\sqrt{5}}$. **Dependent on both previous method marks**	ddM1
	$3\alpha-1=\pm5$	Correct equations (must see ± for this mark but may still score one of the final A marks if ± is missing). May be implied and allow un-simplified.	A1
	$\alpha=2,\ -\frac{4}{3}$	cso	A1, A1
			(6)
			Total 11

7	Π_1: $x+y+z=3$, Π_2: $2x+3y-z=4$		
(a) **Way 1**	$x=\lambda \Rightarrow y=\frac{7}{4}-\frac{3}{4}\lambda$ or $\lambda=\frac{4y-7}{-3}$	M1: Obtains 2 equations connecting x, y or z with λ A1: Correct equations	M1A1
	$z=\frac{5}{4}-\frac{1}{4}\lambda$ or $\lambda=5-4z$	M1: Obtains 3 equations connecting x, y or z with λ A1: Correct equations	M1A1
	$\frac{x}{1}=\frac{7-4y}{3}=\frac{5-4z}{1}(=\lambda)$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
	$y=\lambda \Rightarrow \frac{7-3x}{4}=\frac{y}{1}=\frac{3z-2}{1}$ $\left(\text{or } \frac{7-3x}{4}=y=3z-2\right)$ $z=\lambda \Rightarrow \frac{5-x}{4}=\frac{y+2}{3}=\frac{z}{1}$ (or $=z$)		
			(6)
(a) **Way 2**	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix}=\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$	M1: Attempt vector product of normals A1: Correct vector	M1A1
	$x=0 \Rightarrow y+z=3,\ 3y-z=4$ $\Rightarrow y=\frac{7}{4},\ z=\frac{5}{4} \rightarrow \left(0,\ \frac{7}{4},\ \frac{5}{4}\right)$ NB $y=0$ gives $x=\frac{7}{3}, z=\frac{2}{3}$ $z=0$ gives $x=5,\ y=-2$	M1: Attempt a point on the line A1: Correct point (1, 1, 1) seen frequently	M1A1
	$\frac{x}{-4}=\frac{y-\frac{7}{4}}{3}=\frac{z-\frac{5}{4}}{1}(=\lambda)$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
	or $\frac{x-1}{-4}=\frac{y-1}{3}=\frac{z-1}{1}(=\lambda)$	Equation seen if (1, 1, 1) used	**(6)**
(a) **Way 3**	$x=-\frac{4}{3}y+\frac{7}{3}$	M1: Eliminates 1 variable A1: Correct equation	M1A1
	$x=5-4z$	M1: Eliminates 2nd variable A1: Correct equation	M1A1
	$\frac{x}{1}=-\frac{4}{3}y+\frac{7}{3}=5-4z$	M1: Correct use of cartesian form A1: Correct equation (allow equivalents)	M1A1
			(6)
(b)	$5(-4\lambda)-4\left(\frac{7}{4}+3\lambda\right)+4\left(\frac{5}{4}+\lambda\right)=12$	Substitutes parametric form of L into Π_3	M1
	$\lambda=-\frac{1}{2} \Rightarrow x=...,y=...,z=...$	Solves for λ and attempts coordinates	dM1
	$\left(2,\frac{1}{4},\frac{3}{4}\right)$ or $x=2, y=\frac{1}{4}, z=\frac{3}{4}$ or $\begin{pmatrix} 2 \\ 1/4 \\ 3/4 \end{pmatrix}$	Correct coordinates	A1
			(3)

(b) Way 2	$5x-4.\frac{3}{4}\left(\frac{7}{3}-x\right)+4.\frac{1}{4}(5-x)=12$	Substitutes for y and z in terms of x into Π_3	M1
	$x=2\Rightarrow y=...,z=...$	Solves for x and attempts other coordinates	dM1
	$\left(2,\frac{1}{4},\frac{3}{4}\right)$ or $x=2, y=\frac{1}{4}, z=\frac{3}{4}$ or $\begin{pmatrix}2\\1/4\\3/4\end{pmatrix}$	Correct coordinates	A1

(c)	$\begin{pmatrix}-2\\-\frac{1}{4}\\-\frac{3}{4}\end{pmatrix}\bullet\begin{pmatrix}-4\\3\\1\end{pmatrix}=\sqrt{\frac{37}{8}}\sqrt{26}\cos\theta$	Use scalar product between $\pm$ their $\overrightarrow{OA}$ and direction of their L	M1
	$\frac{13}{2}=\sqrt{\frac{37}{8}}\sqrt{26}\cos\theta\Rightarrow\theta=...$	Evaluate the scalar product and complete to $\theta=$ … (or the supplementary angle) (Check the product if the vectors are incorrect)	dM1
	$\theta=53.6°$	cao	A1
			(3)
			Total 12

Question Number	Scheme		Marks
8(a)	$((2+3\lambda)\mathbf{i}+(1+2\lambda)\mathbf{j}+(-2+\lambda)\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})=19$ $\Rightarrow 2+1+4+3\lambda+2\lambda-2\lambda=19 \Rightarrow \lambda=...$		M1
	Correct dot product leading to value for λ		
	$\lambda=4$	Correct λ	A1
	$(2+3\times"4",1+2\times"4",-2+"4")$	Substitutes their λ to give coordinates	M1
	(14, 9, 2)	Correct coordinates (allow as vector)	A1
			(4)
(b)	$\overline{AB}=2\mathbf{i}+2\mathbf{j}-4\mathbf{k}=2(\mathbf{i}+\mathbf{j}-2\mathbf{k})$ so is perpendicular to plane		M1
	Correct $\overline{AB}$ and conclusion		
	Also B lies on the plane as $(4\mathbf{i}+3\mathbf{j}-6\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})=19$		M1
	Substitutes B into the plane equation and conclusion		
	So coordinates of B are (4, 3, -6)*	Both M's scored with final conclusion	A1*
			(3)
	Alternative		
	$((2+\lambda)\mathbf{i}+(1+\lambda)\mathbf{j}+(-2-2\lambda)\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})=19$ $\Rightarrow 2+1+4+\lambda+\lambda+4\lambda=19 \Rightarrow \lambda=...$		M1
	Correct dot product leading to value for $\lambda\ (=2)$		
	$(2+"2",1+"2",-2-2\times"2")$	Substitutes their λ to give coordinates	M1
	So coordinates of B are (4, 3, -6)*	Both M's scored with final conclusion	A1
(c)	$\overline{OA}'=\overline{OA}+2\overline{AB}$ or $\overline{OB}+\overline{AB}$ (2+4, 1+4, -2-8) or (4+2, 3+2, -6-4)	Correct strategy for finding A'	M1
	(6, 5, -10)	Correct coordinates	A1
			(2)
(d)	**NB require line through their (14, 9, 2) and their (6, 5, -10)**		
	$\pm(14\mathbf{i}+9\mathbf{j}+2\mathbf{k}-(6\mathbf{i}+5\mathbf{j}-10\mathbf{k}))$	Correct attempt at the direction	M1
	$\mathbf{a}=8\mathbf{i}+4\mathbf{j}+12\mathbf{k}$	$\mu(8\mathbf{i}+4\mathbf{j}+12\mathbf{k})$	A1
	$\mathbf{b}=(6\mathbf{i}+5\mathbf{j}-10\mathbf{k})\times(8\mathbf{i}+4\mathbf{j}+12\mathbf{k})$ or $(14\mathbf{i}+9\mathbf{j}+2\mathbf{k})\times(8\mathbf{i}+4\mathbf{j}+12\mathbf{k})$ $=(=100\mathbf{i}-152\mathbf{j}-16\mathbf{k})$		dM1
	Attempt vector product of their $6\mathbf{i}+5\mathbf{j}-10\mathbf{k}$ with their $8\mathbf{i}+4\mathbf{j}+12\mathbf{k}$ **Dependent on the previous M1**		
	$\mathbf{r}\times(2\mathbf{i}+\mathbf{j}+3\mathbf{k})=25\mathbf{i}-38\mathbf{j}-4\mathbf{k}$	$\lambda(\mathbf{r}\times(2\mathbf{i}+\mathbf{j}+3\mathbf{k})=25\mathbf{i}-38\mathbf{j}-4\mathbf{k})$	A1
	Must be in this form for A1 and not just stating **a** and **b**		
			(4)
			Total 13

7(a)	$\begin{pmatrix}0\\3\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\2\end{pmatrix}=8\mathbf{i}-2\mathbf{j}-3\mathbf{k}$	M1: Attempts vector product of two vectors in the plane. Unless there is a full clear method they must achieve two correct components A1: $\pm(8\mathbf{i}-2\mathbf{j}-3\mathbf{k})$ or multiple	M1 A1
	Allow any vector notation throughout this question		**(2)**
(b)	l has direction vector $\pm(2\mathbf{j}+2\mathbf{k})$	Correct direction for l	B1
	$(\cos\alpha \text{ or } \sin\theta =)$ $\left\lvert\frac{"(8\mathbf{i}-2\mathbf{j}-3\mathbf{k})"."(2\mathbf{j}+2\mathbf{k})"}{"\sqrt{8^2+2^2+3^2}"\times"\sqrt{2^2+2^2}"}\right\rvert=\left\lvert\frac{"(8)(0)+(-2)(2)+(-3)(2)"}{"\sqrt{8^2+2^2+3^2}"\times"\sqrt{0^2+2^2+2^2}"}\right\rvert\left(=\left\lvert\frac{-10}{\sqrt{77}\times\sqrt{8}}\right\rvert \text{ or } \left\lvert\frac{-5\sqrt{154}}{154}\right\rvert\right)$ M1: For the scalar product of their normal and direction vector divided by the product of the magnitudes of their vectors. The first expression above oe is sufficient. There must have been a valid attempt at both vectors. Allow copying errors/slips if intention is clear. Modulus not required. A1ft: A correct ft numerical expression with scalar product calculated as shown by second expression or better. Allow a decimal correct to 2sf. Modulus not required. Ignore labelling. Actual decimal is 0.40291148... Implied by awrt 24 or 66 or 114 provided some work and nothing incorrect seen. Allow awrt 0.41, 1.16 or 1.99 if working in radians.		M1 A1ft
	Acute angle between l and P $=90-\alpha=90-66.23968409...$ or $\theta=23.76031591...\Rightarrow 24°$ to the nearest degree	awrt 24 from correct work which could be minimal. Degrees symbol not required. **Mark final answer.**	A1
			(4)
	Note that a vector product could be used: M1: $\left\lvert\frac{\lvert"(8\mathbf{i}-2\mathbf{j}-3\mathbf{k})"\times"(2\mathbf{j}+2\mathbf{k})"\rvert}{"\sqrt{8^2+2^2+3^2}"\times"\sqrt{2^2+2^2}"}\right\rvert$ A1: $\left\lvert\frac{"\sqrt{2^2+16^2+16^2}"}{"\sqrt{8^2+2^2+3^2}"\times"\sqrt{2^2+2^2}"}\right\rvert\left(=\frac{2\sqrt{129}}{\sqrt{77}\sqrt{8}}=0.9152389511...\right)$ The modulus of the numerator is required for any marks		
(c) **Way 1** **Parallel planes**	$(\mathbf{i}+2\mathbf{j}+3\mathbf{k}).("8\mathbf{i}-2\mathbf{j}-3\mathbf{k}")=-5$ or $(6\mathbf{i}-3\mathbf{j}-6\mathbf{k}).("8\mathbf{i}-2\mathbf{j}-3\mathbf{k}")=72$	M1: Finds a value for the scalar product of a position vector of a point in the plane or the given point and their normal. A1: –5 or 72 (or 5 or -72 if normal is in the opposite direction). May be seen as a distance e.g., $\frac{-5}{\sqrt{"77"}}$	M1 A1
	Shortest distance is $\left\lvert\frac{-5-72}{\sqrt{77}}\right\rvert=\frac{77}{\sqrt{77}}$ or $\sqrt{77}$	**d**M1: Having attempted both scalar products, obtains a numerical expression for the distance. Award for $\frac{\pm"5"\pm"72"}{\sqrt{"8"^2+"2"^2+"3"^2}}$ **Dependent on previous M mark.** A1: Correct exact distance. Isw	dM1 A1
			(4)

Question Number	Scheme	Notes	Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\1\\1\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}1\\-1\\3\end{pmatrix},\ \overrightarrow{BC}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 1 & 1\\1 & -1 & 3\end{vmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.) A1: Correct normal vector	M1A1
	$\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=4+14+3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x+7y+z=21$	Cao (oe)	A1
	(a) Alternative		
	$a+2b+3c=d$ $-a+3b+4c=d$ $2a+b+6c=d$	Correct equations	B1
	$a=\frac{4}{21}d,\ b=\frac{1}{3}d,\ c=\frac{1}{21}d$	M1: Solve for a, b and c in terms of d A1: Correct equations	M1A1
	$d=21\Rightarrow a=...,\ b=...,\ c=...$	Obtains values for a, b, c and d	M1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative: Using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ where **b** and **c** are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\3\end{pmatrix}$		M1
	$x=1-2s+t$ $y=2+s-t$ $z=3+s+3t$	Deduce 3 correct equations	A1
	$4x+7y+z=21$	M1: Eliminate s, t A1: Cao	M1A1

	Alternative: Using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ where **b** and **c** are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\3\end{pmatrix}$		M1
	$x=1-2s+t$ $y=2+s-t$ $z=3+s+3t$	Deduce 3 correct equations	A1
	$4x+7y+z=21$	M1: Eliminate s, t A1: Cao	M1A1

(b)	AD□AB × AC	Attempt suitable triple product	M1
	$=\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}k-1\\2\\11\end{pmatrix}=4k-4+14+11$		
	$\therefore\frac{1}{6}(4k+21)=6$	M1: Set $\frac{1}{6}$(their triple product) = 6 A1: Correct equation	dM1A1
	$k=\frac{15}{4}$	Cao (oe)	A1

	(b) Alternative		
	Area ABC $=\frac{1}{2}\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AC}\right\|=\frac{1}{2}\sqrt{6}\sqrt{11}$ D to Π is $\frac{4k+28+14-21}{\sqrt{16+49+1}}$	Attempt area ABC and distance between D and Π	M1
	$\frac{1}{6}\sqrt{6}\sqrt{11}\frac{4k+28+14-21}{\sqrt{16+49+1}}=6$	M1: Set $\frac{1}{3}$(their area x their distance) = 6 A1: Correct equation	dM1A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
			Total 9

Question Number	Scheme	Marks
	Accept **i**, **j**, **k** notation or column vector notation throughout this question	
8(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 5 & -6 \\ -3 & 1 & 2 \end{vmatrix} = \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix}$	B1
		(1)
(b)	$\begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \ldots$ or $\begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \ldots$	M1
	$\mathbf{r}.(8\mathbf{i}+2\mathbf{j}+11\mathbf{k}) = 25$	A1
		(2)
ALT Using another point on the plane	e.g. $\lambda = 0, \mu = 1 \Rightarrow$ point $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ on the plane $\Rightarrow \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \ldots$ or $\Rightarrow \begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \ldots$	M1
	$\mathbf{r}.(8\mathbf{i}+2\mathbf{j}+11\mathbf{k}) = 25$	A1
		(2)
(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 2 & 11 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 13 \\ 3 \\ -10 \end{pmatrix}$ Or Points on the line are (see below) $\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ so direction is $\begin{pmatrix} \frac{52}{3} \\ 4 \\ -\frac{40}{3} \end{pmatrix}$ or any multiple e.g. $\begin{pmatrix} 52 \\ 12 \\ -40 \end{pmatrix}$ etc	M1
	$x = \ldots(0) \Rightarrow \begin{cases} 2y+11z=25 \\ -y+z=7 \end{cases} \Rightarrow z = \ldots(3), y = \ldots(-4)$ $y = \ldots(0) \Rightarrow \begin{cases} 8x+11z=25 \\ x+z=7 \end{cases} \Rightarrow z = \ldots\left(-\frac{31}{3}\right), x = \ldots\left(\frac{52}{3}\right)$ $z = \ldots(0) \Rightarrow \begin{cases} 8x+2y=25 \\ x-y=7 \end{cases} \Rightarrow x = \ldots\left(\frac{39}{10}\right), y = \ldots\left(-\frac{31}{10}\right)$ **Note**: points will have the form $\left(\frac{52+13\alpha}{3}, \alpha, \frac{-31-10\alpha}{3}\right)$	M1

	Common points $(0,-4,3)$ $\left(\frac{52}{3},0,-\frac{31}{3}\right)$ $\left(\frac{39}{10},-\frac{31}{10},0\right)$ $\left(1,-\frac{49}{13},\frac{29}{13}\right)$ $\left(\frac{65}{3},1,-\frac{41}{3}\right)$ $\left(\frac{13}{5},-\frac{17}{5},1\right)$	
	$(\mathbf{r}-(-4\mathbf{j}+3\mathbf{k}))\times(13\mathbf{i}+3\mathbf{j}-10\mathbf{k})=\mathbf{0}$ or $\left(\mathbf{r}-\left(\frac{52}{3}\mathbf{i}-\frac{31}{3}\mathbf{k}\right)\right)\times(13\mathbf{i}+3\mathbf{j}-10\mathbf{k})=\mathbf{0}$ or $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times(13\mathbf{i}+3\mathbf{j}-10\mathbf{k})=\mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)
Alt to find a point and/or direction vector for either M1 mark	Combining cartesian equations together (other eliminations are possible): $\begin{cases}x-y+z=7\\8x+2y+11z=25\end{cases}\Rightarrow 3x-13y=52\Rightarrow y=\frac{3x-52}{13}\quad x=\frac{52+13y}{3}$ $z=7-x+y=\frac{-31-10y}{3}\Rightarrow y=\frac{-3z-31}{10}$ So $\frac{3x-52}{13}=y=\frac{-3z-31}{10}\Rightarrow\frac{x-\frac{52}{3}}{\frac{13}{3}}=\frac{y-0}{1}=\frac{z+\frac{31}{3}}{-\frac{10}{3}}$ so point is $\begin{pmatrix}\frac{52}{3}\\0\\-\frac{31}{3}\end{pmatrix}$ Or Direction is $\begin{pmatrix}\frac{13}{3}\\1\\-\frac{10}{3}\end{pmatrix}$ or any multiple	M1
	point is $\begin{pmatrix}\frac{52}{3}\\0\\-\frac{31}{3}\end{pmatrix}$ and direction is $\begin{pmatrix}\frac{13}{3}\\1\\-\frac{10}{3}\end{pmatrix}$ or any multiple	M1
	$(\mathbf{r}-(-4\mathbf{j}+3\mathbf{k}))\times(13\mathbf{i}+3\mathbf{j}-10\mathbf{k})=\mathbf{0}$ Or $\left(\mathbf{r}-\left(\frac{52}{3}\mathbf{i}-\frac{31}{3}\mathbf{k}\right)\right)\times(13\mathbf{i}+3\mathbf{j}-10\mathbf{k})=\mathbf{0}$ Or $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times(13\mathbf{i}+3\mathbf{j}-10\mathbf{k})=\mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)

(d)	$\pm\left(\begin{pmatrix}3\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right)=\pm\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\13&3&-10\\1&-1&-1\end{vmatrix}=\begin{pmatrix}-13\\3\\-16\end{pmatrix}$ or $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-1&-1\\13&3&-10\end{vmatrix}=\begin{pmatrix}13\\-3\\16\end{pmatrix}$	M1
	$d=\left\lvert\dfrac{(-4\mathbf{j}+3\mathbf{k}-(3\mathbf{i}+2\mathbf{k}))\cdot(-13\mathbf{i}+3\mathbf{j}-16\mathbf{k})}{\sqrt{13^2+3^2+16^2}}\right\rvert=\ldots$	M1
	$d=\dfrac{11}{\sqrt{434}}$	A1
		(4)
Alt using general points on the lines	$\pm\left(\begin{pmatrix}3\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right)=\pm\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1
	Let the general points on the lines be X and Y $\overrightarrow{XY}=\begin{pmatrix}2+\lambda\\1-\lambda\\3-\lambda\end{pmatrix}-\begin{pmatrix}0+13\mu\\-4+3\mu\\3-10\mu\end{pmatrix}=\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\Rightarrow$ $\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\bullet\begin{pmatrix}13\\3\\-10\end{pmatrix}=0$ and $\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-1\end{pmatrix}=0\Rightarrow\begin{cases}41+20\lambda-278\mu=0\\-3+3\lambda-20\mu=0\end{cases}$ $\Rightarrow\lambda=\ldots\left(\dfrac{827}{217}\right),\mu=\ldots\left(\dfrac{183}{434}\right)$	M1
	$\overrightarrow{XY}=\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}=\begin{pmatrix}\frac{143}{434}\\-\frac{33}{434}\\\frac{88}{217}\end{pmatrix}$ $\lvert\overrightarrow{XY}\rvert=\sqrt{\left(\dfrac{143}{434}\right)^2+\left(\dfrac{33}{434}\right)^2+\left(\dfrac{88}{217}\right)^2}=\ldots\left(\dfrac{121}{434}\right)$	M1
	$d=\dfrac{11}{\sqrt{434}}$	A1
		(4)
		Total 10

Question Number	Scheme	Notes	Marks
7(a)	$\frac{x-3}{4}=\frac{y-5}{-2}=\frac{z-4}{7}\Rightarrow \mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-2\\7\end{pmatrix}$	Converts to parametric form. "$\mathbf{r}=$" is not required	M1
	$2x+4y-z=1$ $\Rightarrow 2(3+4\lambda)+4(5-2\lambda)-4-7\lambda=1$ $\Rightarrow \lambda=\ldots(3)\Rightarrow P$ is ...	Correct strategy for finding P. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\ -1,\ 25)$	Correct coordinates. Condone if given as a vector.	A1
			(3)
(a) Way 2	$\frac{x-3}{4}=\frac{y-5}{-2}\Rightarrow x=13-2y$	Uses the Cartesian equation to find x in terms of y	M1
	$2x+4y-z=1\Rightarrow 26-4y+4y-z=1$ $\Rightarrow z=\ldots,\ x=\ldots,\ y=\ldots$	Correct strategy for finding P. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\ -1,\ 25)$	Correct coordinates. Condone if given as a vector.	A1
(b)	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-1\end{pmatrix}=8-8-7=-7$	Applies the scalar product between the direction of l_1 and the normal to the plane	M1
	Examples: $\phi=\cos^{-1}\frac{\pm7}{\sqrt{69}\sqrt{21}}=\ldots\quad \phi=\sin^{-1}\frac{\pm7}{\sqrt{69}\sqrt{21}}=\ldots$ Attempts to find a relevant angle in degrees or radians. **Depends on the first method mark.**		dM1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1
			(3)
(b) Way 2	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\times\begin{pmatrix}2\\4\\-1\end{pmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1	M1
	$\sqrt{26^2+18^2+20^2}=\sqrt{21}\sqrt{69}\sin\alpha$ $\sin\alpha=\frac{10\sqrt{46}}{69}\Rightarrow\alpha=\ldots$	Attempts to find a relevant angle. **Depends on the first method mark.**	dM1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1

(c)	$\mathbf{a}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 4 & -1\\ 4 & -2 & 7\end{vmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1. If no method is seen expect at least 2 correct components.	M1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 13 & -9 & -10\\ 2 & 4 & -1\end{vmatrix}=\begin{pmatrix}49\\-7\\70\end{pmatrix}$	Attempts vector product of "**a**" with normal to Π to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}15\\-1\\25\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Depends on both previous M marks Attempts vector equation using their direction vector and their P	ddM1
		Correct equation or any equivalent correct vector equation	A1
			(5)
(c) Way 2	$\lambda=1\Rightarrow(7,\ 3,\ 11)$ lies on l_1 $\mathbf{r}=\begin{pmatrix}7\\3\\11\end{pmatrix}+t\begin{pmatrix}2\\4\\-1\end{pmatrix}$ $\Rightarrow 2(7+2t)+4(3+4t)-11+t=1$ $t=-\frac{2}{3}\Rightarrow\left(\frac{17}{3},\ \frac{1}{3},\ \frac{35}{3}\right)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix}15\\-1\\25\end{pmatrix}-\frac{1}{3}\begin{pmatrix}17\\1\\35\end{pmatrix}=\frac{1}{3}\begin{pmatrix}28\\-4\\40\end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}15\\-1\\25\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have **r** = and not e.g. $l_2=\ldots$	A1
(c) Way 3	Normal to plane from l_1 $\mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}+t\begin{pmatrix}2\\4\\-1\end{pmatrix}$ $\Rightarrow 2(3+2t)+4(5+4t)-(4-t)=1$ $t=-1\Rightarrow(1,\ 1,\ 5)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix}15\\-1\\25\end{pmatrix}-\begin{pmatrix}1\\1\\5\end{pmatrix}=\begin{pmatrix}14\\-2\\20\end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}1\\1\\5\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have **r** = and not e.g. $l_2=\ldots$	A1
			Total 11