Question Number	Scheme	Notes	Marks
3(a)	$\mathbf{A}=\begin{pmatrix}2&k&2\\2&2&k\\1&2&2\end{pmatrix}$		
	$\lvert\mathbf{A}\rvert=2(4-2k)-k(4-k)+2(4-2)=0$ $\Rightarrow k^2-8k+12=0\Rightarrow k=\ldots$ Attempts det $\mathbf{A}=0$ and solves 3TQ to obtain 2 values for k Note that the usual rules for solving a 3TQ do not need to be applied as long as 2 values for k are obtained. The attempt at the determinant should be a correct expression for their row or column so allow errors only when collecting terms Note that the rule of Sarrus gives $8+k^2+8-4-4k-4k=0$		M1
	$k=2,\ 6$	Correct values.	A1
	Marks for part (a) can only be scored in their attempt at (a) and not recovered from part (b)		
			(2)
(b)	$\begin{pmatrix}2&k&2\\2&2&k\\1&2&2\end{pmatrix}\rightarrow\begin{pmatrix}4-2k&4-k&2\\2k-4&2&4-k\\k^2-4&2k-4&4-2k\end{pmatrix}\rightarrow\begin{pmatrix}4-2k&k-4&2\\4-2k&2&k-4\\k^2-4&4-2k&4-2k\end{pmatrix}$ Applies the correct method to reach at least a matrix of cofactors Should be an attempt at the minors followed by $\begin{pmatrix}+&-&+\\-&+&-\\+&-&+\end{pmatrix}$ If there is any doubt then look for at least 6 correct cofactors		M1
	$\begin{pmatrix}4-2k&k-4&2\\4-2k&2&k-4\\k^2-4&4-2k&4-2k\end{pmatrix}\rightarrow\begin{pmatrix}4-2k&4-2k&k^2-4\\k-4&2&4-2k\\2&k-4&4-2k\end{pmatrix}$ **d**M1: Attempts adjoint matrix by transposing. Dependent on previous mark. A1: Correct adjoint		**d**M1 A1
	$\mathbf{A}^{-1}=\dfrac{1}{k^2-8k+12}\begin{pmatrix}4-2k&4-2k&k^2-4\\k-4&2&4-2k\\2&k-4&4-2k\end{pmatrix}$ Fully correct inverse **or** follow through their incorrect determinant **from part (a)** where their determinant is a function of k		A1ft
	Ignore any labelling of the matrices and allow any type of brackets around the matrices		
			(4)
			Total 6

Question Number	Scheme	Marks
	Accept **i**, **j**, **k** notation or column vector notation throughout this question	
8(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 5 & -6 \\ -3 & 1 & 2 \end{vmatrix} = \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix}$	B1
		(1)
(b)	$\begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \ldots$ or $\begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \ldots$	M1
	$\mathbf{r}.(8\mathbf{i}+2\mathbf{j}+11\mathbf{k}) = 25$	A1
		(2)
ALT Using another point on the plane	e.g. $\lambda = 0, \mu = 1 \Rightarrow$ point $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ on the plane $\Rightarrow \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \ldots$ or $\Rightarrow \begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \ldots$	M1
	$\mathbf{r}.(8\mathbf{i}+2\mathbf{j}+11\mathbf{k}) = 25$	A1
		(2)
(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 2 & 11 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 13 \\ 3 \\ -10 \end{pmatrix}$ Or Points on the line are (see below) $\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ so direction is $\begin{pmatrix} \frac{52}{3} \\ 4 \\ -\frac{40}{3} \end{pmatrix}$ or any multiple e.g. $\begin{pmatrix} 52 \\ 12 \\ -40 \end{pmatrix}$ etc	M1
	$x = \ldots(0) \Rightarrow \begin{cases} 2y+11z=25 \\ -y+z=7 \end{cases} \Rightarrow z = \ldots(3), y = \ldots(-4)$ $y = \ldots(0) \Rightarrow \begin{cases} 8x+11z=25 \\ x+z=7 \end{cases} \Rightarrow z = \ldots\left(-\frac{31}{3}\right), x = \ldots\left(\frac{52}{3}\right)$ $z = \ldots(0) \Rightarrow \begin{cases} 8x+2y=25 \\ x-y=7 \end{cases} \Rightarrow x = \ldots\left(\frac{39}{10}\right), y = \ldots\left(-\frac{31}{10}\right)$ **Note**: points will have the form $\left(\frac{52+13\alpha}{3}, \alpha, \frac{-31-10\alpha}{3}\right)$	M1

	Common points $(0,-4,3)$ $\left(\frac{52}{3},0,-\frac{31}{3}\right)$ $\left(\frac{39}{10},-\frac{31}{10},0\right)$ $\left(1,-\frac{49}{13},\frac{29}{13}\right)$ $\left(\frac{65}{3},1,-\frac{41}{3}\right)$ $\left(\frac{13}{5},-\frac{17}{5},1\right)$	
	$\left(\mathbf{r}-\left(-4\mathbf{j}+3\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ or $\left(\mathbf{r}-\left(\frac{52}{3}\mathbf{i}-\frac{31}{3}\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ or $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)
Alt to find a point and/or direction vector for either M1 mark	Combining cartesian equations together (other eliminations are possible): $\begin{cases}x-y+z=7\\8x+2y+11z=25\end{cases}\Rightarrow 3x-13y=52\Rightarrow y=\frac{3x-52}{13}\quad x=\frac{52+13y}{3}$ $z=7-x+y=\frac{-31-10y}{3}\Rightarrow y=\frac{-3z-31}{10}$ So $\frac{3x-52}{13}=y=\frac{-3z-31}{10}\Rightarrow\frac{x-\frac{52}{3}}{\frac{13}{3}}=\frac{y-0}{1}=\frac{z+\frac{31}{3}}{-\frac{10}{3}}$ so point is $\begin{pmatrix}\frac{52}{3}\\0\\-\frac{31}{3}\end{pmatrix}$ Or Direction is $\begin{pmatrix}\frac{13}{3}\\1\\-\frac{10}{3}\end{pmatrix}$ or any multiple	M1
	point is $\begin{pmatrix}\frac{52}{3}\\0\\-\frac{31}{3}\end{pmatrix}$ and direction is $\begin{pmatrix}\frac{13}{3}\\1\\-\frac{10}{3}\end{pmatrix}$ or any multiple	M1
	$\left(\mathbf{r}-\left(-4\mathbf{j}+3\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or $\left(\mathbf{r}-\left(\frac{52}{3}\mathbf{i}-\frac{31}{3}\mathbf{k}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or $\left(\mathbf{r}-\left(\frac{39}{10}\mathbf{i}-\frac{31}{10}\mathbf{j}\right)\right)\times\left(13\mathbf{i}+3\mathbf{j}-10\mathbf{k}\right)=\mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)

(d)	$\pm\left(\begin{pmatrix}3\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right)=\pm\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\13 & 3 & -10\\1 & -1 & -1\end{vmatrix}=\begin{pmatrix}-13\\3\\-16\end{pmatrix}$ or $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -1 & -1\\13 & 3 & -10\end{vmatrix}=\begin{pmatrix}13\\-3\\16\end{pmatrix}$	M1
	$d=\left\lvert\dfrac{(-4\mathbf{j}+3\mathbf{k}-(3\mathbf{i}+2\mathbf{k}))\cdot(-13\mathbf{i}+3\mathbf{j}-16\mathbf{k})}{\sqrt{13^2+3^2+16^2}}\right\rvert=\ldots$	M1
	$d=\dfrac{11}{\sqrt{434}}$	A1
		(4)
Alt using general points on the lines	$\pm\left(\begin{pmatrix}3\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}\right)=\pm\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1
	Let the general points on the lines be X and Y $\overrightarrow{XY}=\begin{pmatrix}2+\lambda\\1-\lambda\\3-\lambda\end{pmatrix}-\begin{pmatrix}0+13\mu\\-4+3\mu\\3-10\mu\end{pmatrix}=\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\Rightarrow$ $\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\bullet\begin{pmatrix}13\\3\\-10\end{pmatrix}=0$ and $\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-1\end{pmatrix}=0\Rightarrow\begin{cases}41+20\lambda-278\mu=0\\-3+3\lambda-20\mu=0\end{cases}$ $\Rightarrow\lambda=\ldots\left(\dfrac{827}{217}\right),\mu=\ldots\left(\dfrac{183}{434}\right)$	M1
	$\overrightarrow{XY}=\begin{pmatrix}2+\lambda-13\mu\\5-\lambda-3\mu\\-\lambda+10\mu\end{pmatrix}=\begin{pmatrix}\frac{143}{434}\\-\frac{33}{434}\\\frac{88}{217}\end{pmatrix}$ $\lvert\overrightarrow{XY}\rvert=\sqrt{\left(\dfrac{143}{434}\right)^2+\left(\dfrac{33}{434}\right)^2+\left(\dfrac{88}{217}\right)^2}=\ldots\left(\dfrac{121}{434}\right)$	M1
	$d=\dfrac{11}{\sqrt{434}}$	A1
		(4)
		Total 10

Question Number	Scheme	Notes	Marks
3	$\mathbf{M}=\begin{pmatrix}3 & -4 & k\\ 1 & -2 & k\\ 1 & -5 & 5\end{pmatrix}$		
(a)	$\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\mathbf{M}=\begin{vmatrix}3-\lambda & -4 & k\\ 1 & -2-\lambda & k\\ 1 & -5 & 5-\lambda\end{vmatrix}=\begin{vmatrix}0 & -4 & k\\ 1 & -5 & k\\ 1 & -5 & 2\end{vmatrix}$ $(0)+4[2-k]+k[-5+5]$ Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert$ using $\lambda = 3$		M1
	$(0)+4[2-k]+k[-5+5]=0\Rightarrow k=...$ Uses $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ and solves for k		M1
	$k=2$	Cao	A1
			(3)
(b)	$(3-\lambda)[(\lambda+2)(\lambda-5)+10]+4(5-\lambda-2)+2(-5+2+\lambda)=0$ Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ using their value of k		M1
	$\Rightarrow(3-\lambda)[(\lambda+2)(\lambda-5)+12]=0$ $(\lambda+2)(\lambda-5)+12\Rightarrow\lambda^2-3\lambda+2=0\Rightarrow(\lambda-2)(\lambda-1)=0\Rightarrow\lambda=...$ Uses $\lambda = 3$as a factor to obtain and solve a 3TQ to find the other eigenvalues (Alternatively may use calculator to solve $\lambda^3-6\lambda^2+11\lambda-6=0$)		M1
	$\lambda=1, 2$	Correct values	A1
			(3)
(c)	$\begin{pmatrix}3 & -4 & 2\\ 1 & -2 & 2\\ 1 & -5 & 5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=3\begin{pmatrix}x\\ y\\ z\end{pmatrix}\Rightarrow\begin{matrix}3x-4y+2z=3x\\ x-2y+2z=3y\\ x-5y+5z=3z\end{matrix}$	Uses the eigenvalue 3 and their k to form at least 2 equations in x, y and z	M1
	$\alpha\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$ (α a constant)	Any correct eigenvector. Allow any constant multiple of **i** + **j** + 2**k**	A1
	$\frac{1}{\sqrt{6}}\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$	Correct normalised vector	A1
			(3)
			Total 9