Question	Scheme	Marks
4(a)	$\begin{vmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{vmatrix} = 2\begin{vmatrix} 3 & 2 \\ 1 & k \end{vmatrix} - 0\begin{vmatrix} k & 2 \\ -2 & k \end{vmatrix} + (-1)\begin{vmatrix} k & 3 \\ -2 & 1 \end{vmatrix} = 2(3k-2)-(k+6) = \ldots$	**M1**
	$= 6k-4-k-6 = 5k-10\,*$	**A1***
		(2)
(b)	$\mathbf{M}^T = \begin{pmatrix} 2 & k & -2 \\ 0 & 3 & 1 \\ -1 & 2 & k \end{pmatrix}$ or minors $\begin{pmatrix} 3k-2 & k^2+4 & k+6 \\ 1 & 2k-2 & 2 \\ 3 & 4+k & 6 \end{pmatrix}$ or cofactors $\begin{pmatrix} 3k-2 & -k^2-4 & k+6 \\ -1 & 2k-2 & -2 \\ 3 & -4-k & 6 \end{pmatrix}$	**M1**
	Adjugate matrix is $\begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$ (≥ 6 entries correct)	**M1**
	Hence $\mathbf{M}^{-1} = \frac{1}{5k-10}\begin{pmatrix} 3k-2 & -1 & 3 \\ -k^2-4 & 2k-2 & -4-k \\ k+6 & -2 & 6 \end{pmatrix}$	**dM1A1**
		(4)
(c)	Images of A, B and C are $(5, 4k-18, 3k-16)$, $(0, 7-2k, 9-4k)$ and $(0, 4k-2, 8k-14)$	**M1** **A1**
	$(\pm)50 = \frac{1}{6}\begin{vmatrix} 5 & 4k-18 & 3k-16 \\ 0 & 7-2k & 9-4k \\ 0 & 4k-2 & 8k-14 \end{vmatrix} \Rightarrow (\pm)300 = 5(\ldots)(= \;\; 00k-400) \Rightarrow k = \ldots$	**M1**
	$(300 = 200k-400 \Rightarrow)k = \frac{7}{2}$ or $(-300 = 200k-400 \Rightarrow)k = \frac{1}{2}$	**A1**
	$k = \frac{1}{2}$ and $k = \frac{7}{2}$	**A1**
		(5)
Alt method	Using volume scale factor. Attempts $\mathbf{a}.(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 4 & -8 & 3 \\ -2 & 5 & -4 \\ 4 & -6 & 8 \end{vmatrix} = 4(40-24)+8(-16+16)+3(12-20) = \ldots$	**M1**
	Volume of T is $\frac{1}{6}\lvert\mathbf{a}.(\mathbf{b}\times\mathbf{c})\rvert = \left\lvert\frac{1}{6}\begin{vmatrix} 4 & -8 & 3 \\ -2 & 5 & -3 \\ 4 & 6 & -8 \end{vmatrix}\right\rvert = \ldots \frac{20}{3}$	**A1**
	Volume image of $T = \lvert\det \mathbf{M}\rvert \times \frac{20}{3} \Rightarrow \frac{20}{3}\lvert 5k-10\rvert = 50 \Rightarrow k = \ldots$	**M1**

	$\left(\frac{20}{3}(5k-10)=50 \Rightarrow\right) k=\frac{7}{2}$ or $\left(\frac{20}{3}(10-5k)=50 \Rightarrow\right) k=\frac{1}{2}$	**A1**
	$k=\frac{1}{2}$ and $k=\frac{7}{2}$	**A1**
		(5)
		(11 marks)

Question Number	Scheme	Notes	Marks
3(a)	3	Correct value seen in (a)	B1
			(1)
(b)	$\begin{pmatrix}-2 & 5 & 0\\ 5 & 1 & -3\\ 0 & -3 & 6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}8x\\8y\\8z\end{pmatrix} \Rightarrow \begin{matrix}-2x+5y=8x\\5x+y-3z=8y\\-3y+6z=8z\end{matrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\ldots$ Correct method for the eigenvector (making a variable equal to 0 is not a correct method)		M1
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$	Any correct eigenvector	A1
			(2)
(c)	$\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\begin{vmatrix}-2-\lambda & 5 & 0\\ 5 & 1-\lambda & -3\\ 0 & -3 & 6-\lambda\end{vmatrix}=0$ $\Rightarrow(-2-\lambda)\left[(1-\lambda)(6-\lambda)-9\right]-5\left[5(6-\lambda)\right]=0 \Rightarrow \lambda=\ldots$ NB CE is $\lambda^3-5\lambda^2-42\lambda+144=0$ but may only find the constant term		M1
	$\lambda=-6$	Correct third eigenvalue The work for these 2 marks may be seen in (a) – award them Correct third eigenvalue by a different method – send to review	A1
	$\mathbf{D}=\begin{pmatrix}3 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & -6\end{pmatrix}$	Correct **D** following through their third eigenvalue	A1ft
	$\begin{pmatrix}-2 & 5 & 0\\ 5 & 1 & -3\\ 0 & -3 & 6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-6x\\-6y\\-6z\end{pmatrix} \Rightarrow \begin{matrix}-2x+5y=-6x\\5x+y-3z=-6y\\-3y+6z=-6z\end{matrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\left(\begin{pmatrix}-5\\4\\1\end{pmatrix}\right)$ Correct strategy for 3rd eigenvector		M1
	$\mathbf{P}=\begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & -\frac{5}{\sqrt{42}}\\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}}\\ \frac{1}{\sqrt{3}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{42}}\end{pmatrix}$	Fully correct matrix consistent with their **D** May have $\frac{\sqrt{3}}{3}$ etc	A1
			(5)
			Total 8

9	$A(-1, 5, 1),\ B(1, 0, 3),\ C(2, -1, 2),\ D(3, 6, -1)$		
(a)	$\mathbf{AB}=\begin{pmatrix}2\\-5\\2\end{pmatrix},\ \mathbf{AC}=\begin{pmatrix}3\\-6\\1\end{pmatrix},\ \mathbf{AD}=\begin{pmatrix}4\\1\\-2\end{pmatrix}$ $\mathbf{DB}=\begin{pmatrix}2\\6\\-4\end{pmatrix},\ \mathbf{DC}=\begin{pmatrix}1\\7\\-3\end{pmatrix},\ \mathbf{BC}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	Attempts 3 edges of the tetrahedron Any triple with a common vertex Method to be shown or at least 1 correct	M1
	$\begin{vmatrix}2&-5&2\\3&-6&1\\4&1&-2\end{vmatrix}$ or $\begin{pmatrix}2\\-5\\2\end{pmatrix}\cdot\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&-6&1\\4&1&-2\end{vmatrix}$	Attempt appropriate triple product with their edges. (M0 if a vector is obtained)	dM1
	$=\frac{1}{6}(22-50+54)=\frac{13}{3}\ \left(4\frac{1}{3}\text{or } 4.3\text{rec}\right)$	dM1: Completes including $\frac{1}{6}$ (depends on both M marks above) A1: Correct volume (allow equivalents)	ddM1A1
			(4)
	Cartesian method: Find the equation of a plane containing a face of the tetrahedron		M1
	Then find area of triangle and perp height		dM1
	Complete by using Vol $=\frac{1}{3}\times\text{base area}\times\text{height}=\frac{13}{3}$		ddM1A1 (4)
(b)	$\mathbf{AB}\times\mathbf{AC}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&-5&2\\3&-6&1\end{vmatrix}=\begin{pmatrix}7\\4\\3\end{pmatrix}$	M1: Attempt cross product between two sides of *ABC* Min one element correct. A1: Correct normal vector (any multiple)	M1A1
	$\begin{pmatrix}7\\4\\3\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\1\end{pmatrix}(=16)$	Attempt scalar product using their normal vector Answer correct for their vectors or method shown.	dM1
	$7x+4y+3z=16$	Correct equation (any multiple)	A1 **(4)**
(c)	$\mathbf{DT}=\begin{pmatrix}3\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}7\\4\\3\end{pmatrix}$	Attempt parametric form of **DT** using their normal vector	M1
	$7(3+7\lambda)+4(6+4\lambda)+3(-1+3\lambda)=16$ $\Rightarrow\lambda=\ldots$	Substitutes parametric form of **DT** into their plane equation and solves for λ	dM1
	$\lambda=-\frac{13}{37}\Rightarrow T$ is $\left(\frac{20}{37},\frac{170}{37},-\frac{76}{37}\right)$	M1: Uses their value of λ in their **DT** equation. Can be indicated by any coordinate correct for their **DT** and λ A1: Correct exact coordinates Or correct vector $\overrightarrow{OT}$	ddM1A1
			(4)
			Total 12