8. Given that

$$y = e^{3x} \cosh 2x$$

prove by induction that for $n \in \mathbb{N}$

$$\frac{d^n y}{dx^n} = e^{3x}\left(\frac{5^n + 1}{2}\cosh 2x + \frac{5^n - 1}{2}\sinh 2x\right)$$

(6)

DO NOT WRITE IN THIS

2. Determine

(i) $\int \frac{1}{3x^2 + 12x + 24} dx$

(4)

(ii) $\int \frac{1}{\sqrt{27 - 6x - x^2}} dx$

(4)

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1. Relative to a fixed origin O, the points A, B, C and D have coordinates (0, 4, 1), (4, 0, 0), (3, 5, 2) and (2, 2, k) respectively, where k is a constant.

(a) Determine the exact area of triangle ABC.

(3)

(b) Determine in terms of k, the volume of the tetrahedron $ABCD$, simplifying your answer.

(3)

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8. (a) Show that, under the substitution $x = \frac{3}{4}\sinh u$,

$$\int \frac{x^2}{\sqrt{16x^2+9}}\,dx = k\int (\cosh 2u - 1)\,du$$

where k is a constant to be determined.

(6)

(b) Hence show that

$$\int_0^1 \frac{64x^2}{\sqrt{16x^2+9}}\,dx = p + q\ln 3$$

where p and q are rational numbers to be found.

(5)

4. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The line l is a normal to H at the point $P\,(a\sec\theta, b\tan\theta)$, $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$ax\sin\theta + by = (a^2 + b^2)\tan\theta$$

(5)

The line l meets the x-axis at the point Q, and the point M is the midpoint of PQ.

(b) Find the coordinates of M.

(3)

(c) Hence find the cartesian equation of the locus of M as θ varies, giving your answer in the form $y^2 = f(x)$.

(4)

8.
$$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 1$

$$I_n = I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$$

(5)

(b) Hence show that

$$\int_0^{\ln 2} \tanh^4 x \, dx = p + \ln 2$$

where p is a rational number to be found.

(5)

2.

$$\mathbf{T} = \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix} \qquad \mathbf{U} = \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix}$$

where a, b and c are constants.

Given that $\mathbf{TU} = \mathbf{I}$

(a) determine the value of a, the value of b and the value of c

(4)

The transformation represented by the matrix $\mathbf{T}$ transforms the line l_1 to the line l_2

Given that l_2 has equation

$$\frac{x-1}{3} = \frac{y}{-4} = z + 2$$

(b) determine a Cartesian equation for l_1

(4)

5. The hyperbola H has equation $\dfrac{x^2}{25} - \dfrac{y^2}{4} = 1$

The line l has equation $y = mx + c$, where m and c are constants.

Given that l is a tangent to H,

(a) show that $25m^2 = 4 + c^2$

(4)

(b) Hence find the equations of the tangents to H that pass through the point $(1, 2)$.

(5)

(c) Find the coordinates of the point of contact each of these tangents makes with H.

(3)