2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -4 & -3 \\ 0 & -4 & 0 \end{pmatrix}$$

Given that $\mathbf{M}$ has exactly two distinct eigenvalues λ_1 and λ_2 where $\lambda_1 < \lambda_2$

(a) determine a normalised eigenvector corresponding to the eigenvalue λ_1

(6)

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, where μ is a scalar parameter.

The transformation T is represented by $\mathbf{M}$.

The line l_1 is transformed by T to the line l_2

(b) Determine a vector equation for l_2, giving your answer in the form $\mathbf{r} \times \mathbf{b} = \mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are constant vectors.

(3)

4. A non-singular matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 3 & k & 0 \\ k & 2 & 0 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

(a) Find, in terms of k, the inverse of the matrix $\mathbf{M}$.

(5)

The point A is mapped onto the point $(-5, 10, 7)$ by the transformation represented by the matrix

$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(b) Find the coordinates of the point A.

(3)

3.

$$\mathbf{M} = \begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix}$$

Given that $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is an eigenvector of **M**,

(a) determine the corresponding eigenvalue.

(1)

Given that 8 is an eigenvalue of **M**,

(b) determine a corresponding eigenvector.

(2)

(c) Determine a diagonal matrix **D** and an orthogonal matrix **P** such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$$

(5)

3.

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix **M**,

(a) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$,

(2)

(b) show that $k = -7$

(2)

(c) find the other two eigenvalues of the matrix **M**.

(4)

The image of the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ under the transformation represented by **M** is $\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$.

(d) Find the values of the constants p, q and r.

(4)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & -2 & -3 \end{pmatrix}$$

(a) Determine $\mathbf{M}^{-1}$

(3)

The transformation represented by **M** maps the plane Π_1 to the plane Π_2

The point (x, y, z) on Π_1 maps to the point (u, v, w) on Π_2

(b) Determine x, y and z in terms of u, v and w as appropriate.

(3)

The plane Π_1 has equation

$$3x - 7y + 2z = -3$$

(c) Find a Cartesian equation for Π_2

Give your answer in the form $au + bv + cw = d$ where a, b, c and d are integers to be determined.

(2)