3. $$\mathbf{M} = \begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix **M**,

(a) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$,

(2)

(b) show that $k = -7$

(2)

(c) find the other two eigenvalues of the matrix **M**.

(4)

The image of the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ under the transformation represented by **M** is $\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$.

(d) Find the values of the constants p, q and r.

(4)

4. A non-singular matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & k & 0 \\ k & 2 & 0 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

(a) Find, in terms of k, the inverse of the matrix **M**.

(5)

The point A is mapped onto the point $(-5, 10, 7)$ by the transformation represented by the matrix

$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(b) Find the coordinates of the point A.

(3)

4.

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(a) Show that 6 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

(4)

(b) Find a normalised eigenvector corresponding to the eigenvalue 6

(4)

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6:

$$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Determine the value of k for which **A** is singular.

(3)

Given that **A** is non-singular,

(b) determine $\mathbf{A}^{-1}$, giving your answer in simplest form in terms of k.

(4)

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6.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \quad a \neq 1$$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(4)

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **B**.

The equation of l_2 is

$$(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \mathbf{0}$$

(b) Find a vector equation for the line l_1

(4)

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