

3. The ellipse E has equation

$$\frac{x^2}{49} + \frac{y^2}{b^2} = 1$$

where b is a constant and $0 < b < 7$

The eccentricity of the ellipse is e

- (a) Write down, in terms of e only,

- (i) the coordinates of the foci of E
(ii) the equations of the directrices of E

(2)

Given that

- the point $P(x, y)$ lies on E where $x > 0$
- the point S is the focus of E on the positive x -axis
- the line l is the directrix of E which crosses the positive x -axis
- the point M lies on l such that the line through P and M is parallel to the x -axis

- (b) determine an expression for

- (i) PS^2 in terms of e , x and y
(ii) PM^2 in terms of e and x

(2)

- (c) Hence show that

$$b^2 = 49(1 - e^2)$$

(2)

Given that E crosses the y -axis at the points with coordinates $(0, \pm 4\sqrt{3})$

- (d) determine the value of e

(2)

Given that the x coordinate of P is $\frac{7}{2}$

- (e) determine the area of triangle OPM , where O is the origin.

(3)

2. The line with equation $x = 9$ is a directrix of an ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{8} = 1$$

where a is a positive constant.

Find the two possible exact values of the constant a .

(6)

6. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k})$$

and the line l_2 has equation

$$\mathbf{r} = 2\mathbf{i} + s\mathbf{j} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

where s is a constant and λ and μ are scalar parameters.

Given that l_1 and l_2 both lie in a common plane Π_1

(a) show that an equation for Π_1 is $3x + y - z = 3$

(4)

(b) find the value of s .

(1)

The plane Π_2 has equation $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 3$

(c) Find an equation for the line of intersection of Π_1 and Π_2

(4)

(d) Find the acute angle between Π_1 and Π_2 giving your answer in degrees to 3 significant figures.

(4)

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2. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

(i) $\cosh 2x \equiv 2 \cosh^2 x - 1$

(ii) $\sinh 2x \equiv 2 \sinh x \cosh x$

(4)

(b) Solve the equation

$$\cosh 2x - 7 \cosh x = -7$$

giving your answers as exact logarithms.

(5)

2. An ellipse has equation

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

The point P lies on the ellipse and has coordinates $(5 \cos \theta, 2 \sin \theta)$, $0 < \theta < \frac{\pi}{2}$

The line L is a normal to the ellipse at the point P .

(a) Show that an equation for L is

$$5x \sin \theta - 2y \cos \theta = 21 \sin \theta \cos \theta$$

(5)

Given that the line L crosses the y -axis at the point Q and that M is the midpoint of PQ ,

(b) find the exact area of triangle OPM , where O is the origin, giving your answer as a multiple of $\sin 2\theta$

(6)

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