2.

$$\mathbf{T} = \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix} \qquad \mathbf{U} = \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix}$$

where a, b and c are constants.

Given that $\mathbf{TU} = \mathbf{I}$

(a) determine the value of a, the value of b and the value of c

(4)

The transformation represented by the matrix $\mathbf{T}$ transforms the line l_1 to the line l_2

Given that l_2 has equation

$$\frac{x-1}{3} = \frac{y}{-4} = z + 2$$

(b) determine a Cartesian equation for l_1

(4)

8. The plane Π_1 has equation

$$x - 5y + 3z = 11$$

The plane Π_2 has equation

$$3x - 2y + 2z = 7$$

The planes Π_1 and Π_2 intersect in the line l.

(a) Find a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(5)

The point $P(2, 0, 3)$ lies on Π_1

The line m, which passes through P, is parallel to l.

The point $Q(3, 2, 1)$ lies on Π_2

The line n, which passes through Q, is also parallel to l.

(b) Find, in exact simplified form, the shortest distance between m and n.

(5)

6. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l passes through the point $A(1, 7, 3)$ and meets Π at the point $B(-1, 5, 2)$

The acute angle between Π and l is α

(b) Determine the value of α to the nearest degree.

(4)

(c) Determine the exact shortest distance from A to Π

(4)

9. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

where s and t are scalar parameters.

(a) Determine a Cartesian equation for Π_1

(3)

The plane Π_2 has vector equation $\mathbf{r}.\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 1$

(b) Determine a vector equation for the line of intersection of Π_1 and Π_2

Give your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(4)

The plane Π_3 has Cartesian equation $4x - 3y - z = 0$

(c) Use the answer to part (b) to determine the coordinates of the point of intersection of Π_1, Π_2 and Π_3

(3)

7. The plane Π_1 contains the point $(3, 3, -2)$ and the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+1}{4}$

(a) Show that a cartesian equation of the plane Π_1 is

$$3x - 10y - 4z = -13$$

(5)

The plane Π_2 is parallel to the plane Π_1

The point $(\alpha, 1, 1)$, where α is a constant, lies in Π_2

Given that the shortest distance between the planes Π_1 and Π_2 is $\frac{1}{\sqrt{5}}$

(b) find the possible values of α.

(6)

7. The plane Π_1 has equation $x + y + z = 3$ and the plane Π_2 has equation $2x + 3y - z = 4$

The planes Π_1 and Π_2 intersect in the line L.

(a) Find a cartesian equation for the line L.

(6)

The plane Π_3 has equation

$$\mathbf{r} \cdot \begin{pmatrix} 5 \\ -4 \\ 4 \end{pmatrix} = 12$$

The line L meets the plane Π_3 at the point A.

(b) Find the coordinates of A.

(3)

(c) Find the acute angle between $\overrightarrow{OA}$ and the line L, where O is the origin.
Give your answer in degrees to one decimal place.

(3)

8. The line l has equation

$$\mathbf{r} = (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + \lambda(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}), \text{ where } \lambda \text{ is a scalar parameter,}$$

and the plane Π has equation

$$\mathbf{r}.(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 19$$

(a) Find the coordinates of the point of intersection of l and Π.

(4)

The perpendicular to Π from the point A (2, 1, –2) meets Π at the point B.

(b) Verify that the coordinates of B are (4, 3, –6).

(3)

The point A (2, 1, –2) is reflected in the plane Π to give the image point A'.

(c) Find the coordinates of the point A'.

(2)

(d) Find an equation for the line obtained by reflecting the line l in the plane Π, giving your answer in the form

$$\mathbf{r} \times \mathbf{a} = \mathbf{b},$$

where $\mathbf{a}$ and $\mathbf{b}$ are vectors to be found.

(4)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l meets Π at the point (1, 2, 3) and passes through the point (1, 0, 1)

(b) Determine the size of the acute angle between Π and l

Give your answer to the nearest degree.

(4)

(c) Determine the shortest distance between Π and the point (6, −3, −6)

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

6. The coordinates of the points A, B and C relative to a fixed origin O are $(1, 2, 3)$, $(-1, 3, 4)$ and $(2, 1, 6)$ respectively. The plane Π contains the points A, B and C.

(a) Find a cartesian equation of the plane Π.

(5)

The point D has coordinates $(k, 4, 14)$ where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

8: The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 5 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine $(7\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

(1)

(b) Hence show that the equation of Π_1 can be written in the form

$$\mathbf{r}.(8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = p$$

where p is a constant to be determined.

(2)

Given that

- the plane Π_2 has equation $x - y + z = 7$
- the planes Π_1 and Π_2 intersect in the line l_1

(c) determine an equation for l_1 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(3)

Given also that

- the point A has coordinates $(2, 1, 3)$
- the point B has coordinates $(3, 0, 2)$
- the line l_2 passes through A and B

(d) determine the shortest distance between l_1 and l_2

(4)

blank

DO NOT WRITE IN THIS AREA

7. The line l_1 has equation

$$\frac{x-3}{4} = \frac{y-5}{-2} = \frac{z-4}{7}$$

The plane Π has equation

$$2x + 4y - z = 1$$

The line l_1 intersects the plane Π at the point P

(a) Determine the coordinates of P

(3)

The acute angle between l_1 and Π is θ degrees.

(b) Determine, to one decimal place, the value of θ

(3)

The line l_2 lies in Π and passes through P

Given that the acute angle between l_1 and l_2 is also θ degrees,

(c) determine a vector equation for l_2

(5)

DO NOT WRITE IN THIS AREA