3.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(b) find $\mathbf{A}^{-1}$, giving your answer in terms of k.

(4)

8: The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 5 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine $(7\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

(1)

(b) Hence show that the equation of Π_1 can be written in the form

$$\mathbf{r}.(8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = p$$

where p is a constant to be determined.

(2)

Given that

- the plane Π_2 has equation $x - y + z = 7$
- the planes Π_1 and Π_2 intersect in the line l_1

(c) determine an equation for l_1 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(3)

Given also that

- the point A has coordinates (2, 1, 3)
- the point B has coordinates (3, 0, 2)
- the line l_2 passes through A and B

(d) determine the shortest distance between l_1 and l_2

(4)

3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that 3 is an eigenvalue of **M**,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of **M**.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)