4. The matrix $\mathbf{M}$ is given by

$$\begin{pmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{pmatrix}$$

(a) Show that $\det \mathbf{M} = 5k - 10$

(2)

Given that $k \neq 2$

(b) find $\mathbf{M}^{-1}$ in terms of k.

(4)

The points $O(0, 0, 0)$, $A(4, -8, 3)$, $B(-2, 5, -4)$ and $C(4, -6, 8)$ are the vertices of a tetrahedron T.

The transformation represented by matrix $\mathbf{M}$ transforms T to a tetrahedron with volume 50

(c) Determine the possible values of k.

(5)

3.

$$\mathbf{M} = \begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix}$$

Given that $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is an eigenvector of $\mathbf{M}$,

(a) determine the corresponding eigenvalue.

(1)

Given that 8 is an eigenvalue of $\mathbf{M}$,

(b) determine a corresponding eigenvector.

(2)

(c) Determine a diagonal matrix $\mathbf{D}$ and an orthogonal matrix $\mathbf{P}$ such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$$

(5)

9 With respect to a fixed origin O, the points $A(-1, 5, 1)$, $B(1, 0, 3)$, $C(2, -1, 2)$ and $D(3, 6, -1)$ are the vertices of a tetrahedron.

(a) Find the volume of the tetrahedron $ABCD$.

(4)

The plane Π contains the points A, B and C.

(b) Find a cartesian equation of Π.

(4)

The point T lies on the plane Π.

The line DT is perpendicular to Π.

(c) Find the exact coordinates of the point T.

(4)