7.

$$y = \arccos(\operatorname{sech} x) \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \operatorname{sech} x$$

(3)

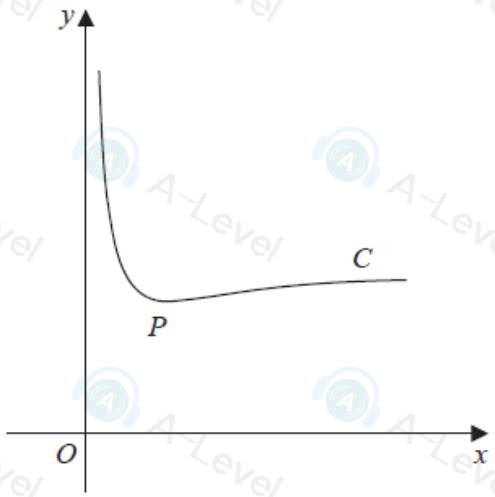


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \arccos(\operatorname{sech} x) + \coth x \qquad x > 0$$

The point P is a minimum turning point of C

(b) Show that the x coordinate of P is $\ln\left(q + \sqrt{q}\right)$ where $q = \frac{1}{2}\left(1 + \sqrt{k}\right)$ and k is an integer to be determined.

(6)

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8. The plane Π_1 has equation

$$x - 5y + 3z = 11$$

The plane Π_2 has equation

$$3x - 2y + 2z = 7$$

The planes Π_1 and Π_2 intersect in the line l.

(a) Find a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(5)

The point $P(2, 0, 3)$ lies on Π_1

The line m, which passes through P, is parallel to l.

The point $Q(3, 2, 1)$ lies on Π_2

The line n, which passes through Q, is also parallel to l.

(b) Find, in exact simplified form, the shortest distance between m and n.

(5)

6. The coordinates of the points A, B and C relative to a fixed origin O are $(1, 2, 3)$, $(-1, 3, 4)$ and $(2, 1, 6)$ respectively. The plane Π contains the points A, B and C.

(a) Find a cartesian equation of the plane Π.

(5)

The point D has coordinates $(k, 4, 14)$ where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

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9. The ellipse E has equation

$$x^2 + 9y^2 = 9$$

The foci of E are F_1 and F_2

(a) (i) Determine the coordinates of F_1 and the coordinates of F_2

(ii) Write down the equation of each of the directrices of E

(4)

The point P lies on the ellipse.

(b) Show that $|PF_1| + |PF_2| = 6$

(3)

The straight line through P with equation $y = 2x + c$ meets E again at the point Q

The point M is the midpoint of PQ

(c) Show that as P varies the locus of M is a straight line passing through the origin.

(6)

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix} \text{ where } k \text{ is a constant}$$

Given that 3 is an eigenvalue of $\mathbf{M}$,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of $\mathbf{M}$.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)

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