4.

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(a) Show that 6 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

(4)

(b) Find a normalised eigenvector corresponding to the eigenvalue 6

(4)

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5.

$$\mathbf{M} = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

Given that 8 is an eigenvalue of **M**

(a) determine an eigenvector corresponding to the eigenvalue 8

(2)

(b) Determine the other two eigenvalues of **M**.

(3)

(c) Hence find an orthogonal matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{MP} = \mathbf{D}$

(4)

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3. $\mathbf{M} = \begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}$, where k is a constant.

Given that $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix $\mathbf{M}$,

(a) find the eigenvalue of $\mathbf{M}$ corresponding to $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$,

(2)

(b) show that $k = -7$

(2)

(c) find the other two eigenvalues of the matrix $\mathbf{M}$.

(4)

The image of the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ under the transformation represented by $\mathbf{M}$ is $\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$.

(d) Find the values of the constants p, q and r.

(4)

4. $\mathbf{M} = \begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}$

where k is a constant.

Given that 3 is an eigenvalue of $\mathbf{M}$,

(a) determine the possible values of k.

(3)

Given that $k < 0$

(b) determine the other eigenvalues of $\mathbf{M}$.

(3)

(c) Determine a normalised eigenvector corresponding to the eigenvalue 3

(3)

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3: $\mathbf{M} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix}$ where a and b are constants

Given that $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ is an eigenvector of $\mathbf{M}$,

(a) determine the corresponding eigenvalue.

(2)

(b) Hence determine the value of a and the value of b.

(2)

(c) Determine

(i) the other eigenvalues of $\mathbf{M}$,

(3)

(ii) eigenvectors which correspond to these eigenvalues.

(3)

3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix}$$ where k is a constant

Given that 3 is an eigenvalue of $\mathbf{M}$,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of $\mathbf{M}$.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)

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4.

$$\mathbf{M} = \begin{pmatrix} 0 & -1 & 3 \\ -1 & 4 & -1 \\ 3 & -1 & 0 \end{pmatrix}$$

Given that $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$

(a) determine its corresponding eigenvalue.

(2)

Given that -3 is an eigenvalue of $\mathbf{M}$

(b) determine a corresponding eigenvector.

(2)

Hence, given that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is also an eigenvector of $\mathbf{M}$

(c) determine a diagonal matrix $\mathbf{D}$ and an orthogonal matrix $\mathbf{P}$ such that $\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$

(4)

5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix}$$ where a is a positive constant

(a) Determine the exact value of a for which the matrix $\mathbf{A}$ is singular.

(2)

Given that 2 is an eigenvalue of $\mathbf{A}$

(b) determine

(i) the value of a

(ii) the other two eigenvalues of $\mathbf{A}$

(5)

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(c) Determine a normalised eigenvector for each of the other eigenvalues of $\mathbf{A}$

(5)

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6. $\mathbf{A} = \begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$

(a) Show that $\mathbf{A}$ is non-singular for all real values of x.

(4)

(b) Determine, in terms of x, $\mathbf{A}^{-1}$

(4)

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