Leave blank

1. A truck of mass 2400 kg is pulling a trailer of mass M kg along a straight horizontal road. The tow bar, connecting the truck to the trailer, is horizontal and parallel to the direction of motion. The tow bar is modelled as being light and inextensible. The resistance forces acting on the truck and the trailer are constant and of magnitude 400 N and 200 N respectively. The acceleration of the truck is 0.5 m s^{-2} and the tension in the tow bar is 600 N.

(a) Find the magnitude of the driving force of the truck.

(3)

(b) Find the value of M.

(3)

(c) Explain how you have used the fact that the tow bar is inextensible in your calculations.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

4.

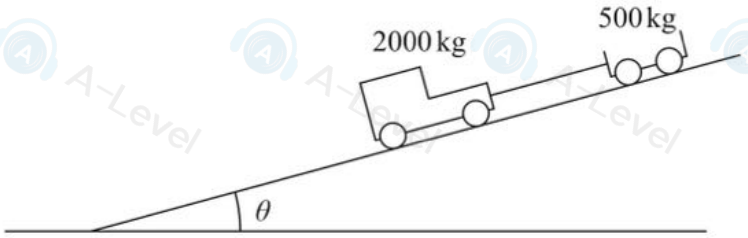

Figure 3

A railway truck of mass 500 kg is pushed up a straight track by a railway engine of mass 2000 kg. The track is inclined to the horizontal at an angle θ, where $\sin\theta = \frac{1}{14}$, as shown in Figure 3. The engine produces a constant driving force of magnitude 3050 N. The truck experiences a constant resistance to motion of magnitude 100 N and the engine experiences a constant resistance to motion of magnitude 200 N. The engine and the truck are connected by a coupling which is modelled as a light rod that is parallel to the track.

Find

(i) the acceleration of the system,

(ii) the magnitude of the force exerted on the truck by the coupling.

(8)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

7.

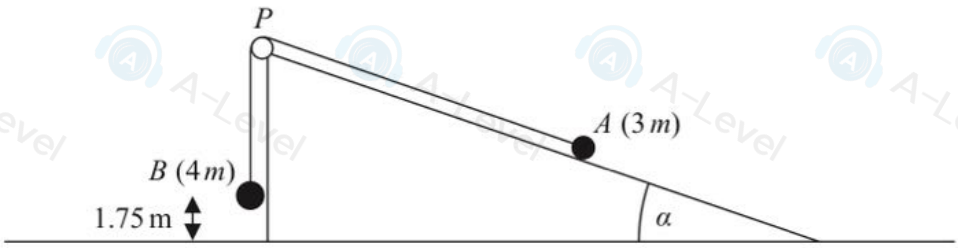

Figure 2

Figure 2 shows two particles A and B, of masses $3m$ and $4m$ respectively, attached to the ends of a light inextensible string. Initially A is held at rest on the surface of a fixed rough inclined plane. The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth light pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely and is vertically below P. The system is released from rest with the string taut and with B at a height of 1.75 m above the ground. In the subsequent motion, A does not hit the pulley.

For the period before B hits the ground,

(a) write down an equation of motion for each particle.

(4)

(b) Hence show that the acceleration of B is $\frac{8}{35}g$.

(5)

(c) Explain how you have used the fact that the string is inextensible in your calculation.

(1)

When B hits the ground, B does not rebound and comes immediately to rest.

(d) Find the distance travelled by A from the instant when the system is released to the instant when A first comes to rest.

(7)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

8.

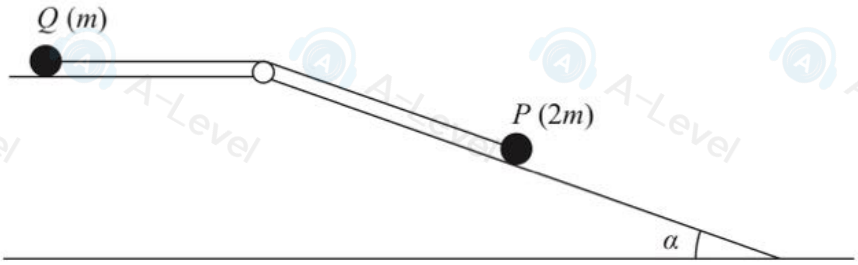

Figure 4

Two particles, P and Q, with masses $2m$ and m respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P is on the surface of a smooth inclined plane. The top of the plane coincides with the edge of the table. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$, as shown in Figure 4. The string lies in a vertical plane containing the pulley and a line of greatest slope of the plane. The coefficient of friction between Q and the table is $\frac{1}{2}$. Particle Q is released from rest with the string taut and P begins to slide down the plane.

(a) By writing down an equation of motion for each particle,

(i) find the initial acceleration of the system,

(ii) find the tension in the string.

(10)

Suppose now that the coefficient of friction between Q and the table is μ and when Q is released it remains at rest.

(b) Find the smallest possible value of μ.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

8.

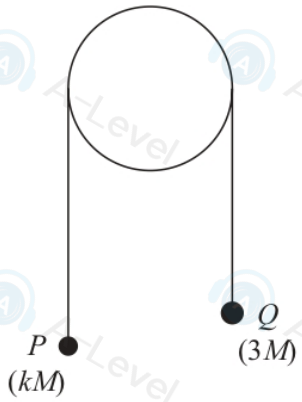

Figure 5

Two small balls, P and Q, have masses kM and $3M$ respectively, where $k < 3$

The balls are attached to the ends of a light inextensible string that passes over a fixed light smooth pulley.

The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5.

The system is released from rest and, in the subsequent motion, P moves with an acceleration of magnitude $\frac{1}{5}g$

The balls are modelled as particles.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of k.

(3)

Given that $M = 0.5\,\text{kg}$,

(c) find the magnitude of the force exerted on the pulley by the string while Q is moving downwards.

(3)

At the instant when the system is released, P is more than 2.5 m from the pulley and Q is 2.5 m above horizontal ground.
After hitting the ground, Q rebounds with a speed of $0.4\,\text{m s}^{-1}$

(d) Find the magnitude of the impulse received by Q when it hits the ground.

(5)

In the subsequent motion, P does not hit the pulley.

(e) Find the total time from when the balls are released until P first comes to rest.

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Leave blank

8.

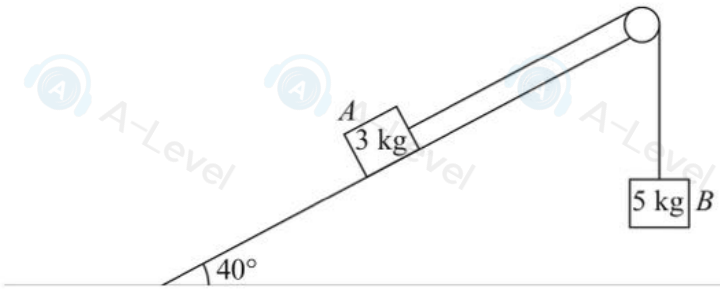

Figure 4

One end of a light inextensible string is attached to a block A of mass 3 kg. Block A is held at rest on a smooth fixed plane. The plane is inclined at 40° to the horizontal ground. The string lies along a line of greatest slope of the plane and passes over a small smooth pulley which is fixed at the top of the plane. The other end of the string is attached to a block B of mass 5 kg. Block B hangs freely at rest below the pulley, as shown in Figure 4. The system is released from rest with the string taut.

By modelling the two blocks as particles,

(a) find the tension in the string as B descends.

(6)

After falling for 1.5 s, block B hits the ground and is immediately brought to rest. In its subsequent motion, A does not reach the pulley.

(b) Find the speed of B at the instant it hits the ground.

(3)

(c) Find the total distance moved up the plane by A before it comes to instantaneous rest.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

6.

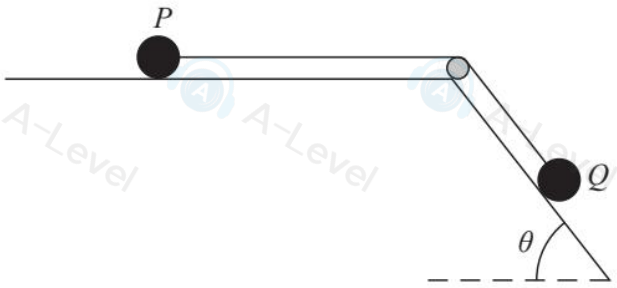

Figure 2

Two particles P and Q have masses 0.1 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a rough horizontal table. The string lies along the table and passes over a small smooth pulley which is fixed to the edge of the table. Particle Q is at rest on a smooth plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{4}{3}$

The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 2. Particle P is released from rest with the string taut. During the first 0.5 s of the motion P does not reach the pulley and Q moves 0.75 m down the plane.

(a) Find the tension in the string during the first 0.5 s of the motion.

(6)

(b) Find the coefficient of friction between P and the table.

(5)

Leave blank

7.

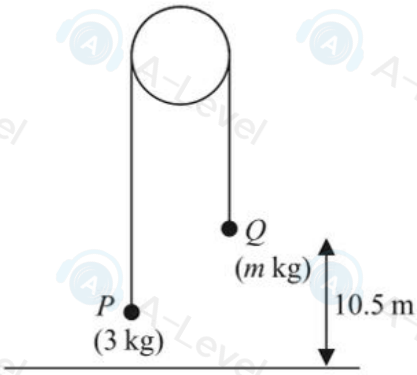

Figure 5

Two particles P and Q have masses 3 kg and m kg respectively ($m > 3$). The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical. The particle Q is at a height of 10.5 m above the horizontal ground, as shown in Figure 5. The system is released from rest and Q moves downwards. In the subsequent motion P does not reach the pulley. After the system is released, the tension in the string is 33.6 N.

(a) Show that the magnitude of the acceleration of P is 1.4 m s^{-2}.

(3)

(b) Find the value of m.

(3)

The system is released from rest at time $t = 0$. At time T_1 seconds after release, Q strikes the ground and does not rebound. The string goes slack and P continues to move upwards.

(c) Find the value of T_1

(3)

At time T_2 seconds after release, P comes to instantaneous rest.

(d) Find the value of T_2

(3)

At time T_3 seconds after release ($T_3 > T_1$) the string becomes taut again.

(e) Sketch a velocity-time graph for the motion of P in the interval $0 \leqslant t \leqslant T_3$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA