Question Number	Scheme	Marks
3		
(a)	$mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = mg\sin\alpha - \dfrac{1}{3}mx^2$	M1A1
	$\dfrac{1}{2}v^2 = xg\sin\alpha - \dfrac{1}{9}x^3 \ (+c)$	DM1A1
	$x = 2 \ \ \dfrac{1}{2}v^2 = 2g\sin\alpha - \dfrac{8}{9}$	DM1
	$(v = 3.728...)$	
	$v = 3.7$ or $3.73 \ \left(\mathrm{m\,s^{-1}}\right)$	A1cso (6)
ALT	By energy:	
	$mg\sin\alpha x = \int \dfrac{1}{3}mx^2\mathrm{d}x + \dfrac{1}{2}mv^2$	M1A1
	$xg\sin\alpha = \dfrac{1}{9}x^3 + \dfrac{1}{2}v^2 \ (+c)$	DM1A1
	$x = 2 \ \ \dfrac{1}{2}v^2 = 2g\sin\alpha - \dfrac{8}{9}$	DM1
	$v = 3.7$ or $3.73 \ \left(\mathrm{m\,s^{-1}}\right)$	A1
(b)	$v = 0 \Rightarrow x^2 = 9g\sin\alpha = 9g \times \dfrac{2}{5} \ (x \neq 0)$	
	$x = 5.939... \Rightarrow OA = 5.9$ or $5.94 \ (\mathrm{m})$	M1A1 (2)
		[8]

Question Number	Scheme	Marks
5(a)	$0.8v\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{4}{(x+1)^3}$	M1A1
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{5}{(x+1)^3}$	
	$\dfrac{1}{2}v^2=-\dfrac{5}{2(x+1)^2}\ (+c)$	dM1A1
	$(t=0,v=0,x=0)\Rightarrow c=\dfrac{5}{2}$	ddM1
	$v^2=5\left(1-\dfrac{1}{(x+1)^2}\right)=5\left(\dfrac{(x+1)^2-1}{(x+1)^2}\right)$ *	A1cso (6)
(b)	$v=\dfrac{\mathrm{d}x}{\mathrm{d}t}=\sqrt{5\left(\dfrac{(x+1)^2-1}{(x+1)^2}\right)}$	M1
	$\int\dfrac{(x+1)}{\sqrt{(x+1)^2-1}}\mathrm{d}x=\int\sqrt{5}\,\mathrm{d}t$	M1A1
	$\left((x+1)^2-1\right)^{\frac{1}{2}}=\sqrt{5}t\ (+k)$	M1A1
	$t=0,x=0\Rightarrow k=0$	
	$(x+1)^2-1=5t^2\qquad x=\sqrt{5t^2+1}-1$	
	Dist $=\sqrt{81}-1-\left(\sqrt{21}-1\right)=9-\sqrt{21}$ or 4.417 m (accept 4.4 or better)	M1A1cso(7) **[13]**

(a)M1	Attempt an equation of motion with acceleration $=v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. Can be implied by subsequent work.
A1	Fully correct equation. Can be implied by subsequent work.
dM1	Attempt the necessary integration. $\dfrac{k}{(x+1)^3}\rightarrow\pm\dfrac{k'}{(x+1)^2}$ Depends on the first M mark
A1	Correct integration (not ft). Constant may be omitted.
ddM1	Use the given initial conditions to obtain a value for the constant. Depends on both preceding M marks. Initial conditions need not be shown explicitly.
A1cso	Reach the **given** answer in the form shown in the question. No errors in the work.

Question Number	Scheme	Marks
3(a)	$v = \dfrac{50}{2x+3}$	
	$\dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt}$	M1
	$= \dfrac{-100}{(2x+3)^2} \times \dfrac{50}{2x+3} \quad \left(= \dfrac{-5000}{(2x+3)^3} \right)$	DM1A1
	$x = 12 \quad \dfrac{dv}{dt} = -\dfrac{5000}{27^3} = -0.2540... = -0.25$ or $-0.254\ \text{m s}^{-2}$	M1
	deceleration $= 0.25\ (\text{m s}^{-2})$ or better	A1 (5)
(b)	$v = \dfrac{dx}{dt} = \dfrac{50}{2x+3}$	M1
	$\int (2x+3)\,dx = \int 50\,dt$	
	$x^2 + 3x = 50t \ (+c)$	M1A1
	$t = 1,\ x = 4 \Rightarrow 28 = 50 + c,\ c = -22$	A1
	$x = 12 \Rightarrow 50t = 12^2 + 36 + 22 \quad t = \dfrac{202}{50} = 4.04$ (accept 4.0)	A1 (5)
		[10]

Notes

(a)

M1 Uses chain rule of the form $\dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt}$ or $\dfrac{d(\frac{1}{2}v^2)}{dx}$

Note, $\dfrac{1}{2}v^2 = \dfrac{1250}{(2x+3)^2} \Rightarrow \text{acc} = \dfrac{d(\frac{1}{2}v^2)}{dx} = -\dfrac{2500}{(2x+3)^3} \times 2$

However, **M0** for acc $= \dfrac{1}{2}v^2$

DM1 Differentiate v wrt x

A1 Correct differentiation.

M1 Sub $x = 12$ into their expression for acceleration to obtain the deceleration. Must have attempted to differentiate.

A1 Correct deceleration – must be positive

(b)

M1 Use $v = \dfrac{dx}{dt}$

M1 Attempt at integration

A1 Correct integration but c may be missing

A1 Use $t = 1,\ x = 4$ to obtain the correct value of c for their correct integration

A1 Sub $x = 12$ to obtain the correct value of t

Question Number	Scheme	Marks
3(a)	Correct method to form a differential equation in v and x, separate the variables and integrate to the form $\frac{1}{2}v^2 = Ax^2 + Bx \quad (+C)$ o.e.	M1
	$\left(v\frac{dv}{dx} = 4x - 2 \Rightarrow\right) \quad \frac{1}{2}v^2 = 2x^2 - 2x \quad (+C)$	A1
	$\frac{1}{2}v^2 = 2x^2 - 2x + \frac{1}{2}$ o.e	A1
	$v = 1 - 2x$	A1
		(4)
3(b) ALT1	Method to form a differential equation in x and t, separate the variables and integrate to the form $D\ln(1-2x) = t \quad (+C)$ o.e.	M1
	$\left(\int \frac{1}{1-2x}dx = \int dt \Rightarrow\right) \quad -\frac{1}{2}\ln(1-2x) = t \ (+C)$	A1
	$(t = 1, x = 0 \Rightarrow C = -1)$ $-\frac{1}{2}\ln(1-2x) = t - 1$ o.e.	A1
	Complete method to obtain v in terms of t For example • $-\frac{1}{2}\ln(v) = t - 1 \ (+C) \Rightarrow v = \ldots$ in terms of t • $x = \frac{1}{2}(1 - e^{2-2t})$ then differentiate wrt t to obtain $v = \ldots$ in terms of t	dM1
	$(v =) e^{2-2t}$	A1
		(5)
3(b) ALT2	Method to form a differential equation in v and t, separate the variables and integrate to the form $\ln v = Et \ (+C)$ o.e.	M1
	$\left(\int \frac{1}{v}dv = -2\int dt \Rightarrow\right) \quad \ln v = -2t \ (+C)$	A1
	$t \ (t = 1, v = 1 \Rightarrow C = 2)$ $\ln(v) = 2 - 2t$ o.e.	A1
	Obtain v in terms of t	dM1
	$v = e^{2-2t}$	A1
		(5)
		(9)
Notes for question 3		
(a)		

Question Number	Scheme	Marks
3 (a)	$0.4\dfrac{\mathrm{d}v}{\mathrm{d}t}=-\dfrac{8}{(t+4)^2}$	M1
	$v=\int-\dfrac{20}{(t+4)^2}\,\mathrm{d}t$	
	$v=\dfrac{20}{(t+4)}\ (+c)$	DM1A1
	$t=0,\ v=10\Rightarrow\ 10=5+c,\ c=5$	DM1
	$v=\dfrac{20}{(t+4)}+5$ *	A1cso (5)
(b)	$x=\int\dfrac{20}{(t+4)}+5\ \mathrm{d}t$	
	$x=20\ln(t+4)+5t+c$	M1A1
	$t=0,\ x=0\Rightarrow 0=20\ln 4+c \quad c=-20\ln 4$	A1
	$v=6\quad\dfrac{20}{t+4}=1\quad t=16$	B1
	$x=20\ln 20+80-20\ln 4=80+20\ln 5$	
	$a=80,\ b=20$	A1cao (5) **[10]**

(a)
M1 0.4 or m for the mass for the first 4 marks
Form an equation of motion with the acceleration in form shown. Minus sign may be missing.
DM1 Attempt the integration
A1 Correct integration, including correct sign. Constant not needed
DM1 Use $t=0,\ v=10$ to obtain a value for c
A1cso Correct answer only, no errors seen

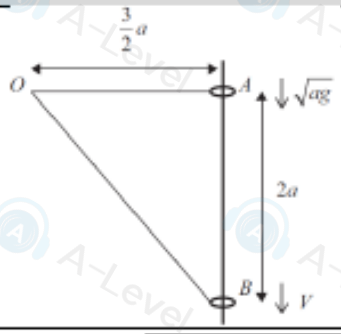
(b)
M1 Integrate the **given** expression for v to obtain an expression for x. Constant not needed.
A1 Correct integration, constant not needed
A1 Obtain the correct constant
B1 Correct value for t when $v=6$
A1cao Correct values for a and b Need not be shown explicitly.

By definite integration:

(a) M1 DM1 as above A1 as above but limits not needed here
DM1 Substitute correct limits A1 cao no errors seen

(b) M1A1 as above but limits not needed here
A1 Substitute lower limit (0) to obtain $-20\ln 4$
B1A1 as above

Question Number	Scheme	Marks
3(a)	$v\frac{dv}{dx}=\frac{3\sqrt{x+1}}{4}$	M1
	$\frac{1}{2}v^2=\frac{1}{2}(x+1)^{\frac{3}{2}}\ (+C)$	M1A1
	$x=15, v=8 \Rightarrow C=0$ so $v=(x+1)^{\frac{3}{4}}$ *	A1* (4)
3(b)	$\frac{dx}{dt}=(x+1)^{\frac{3}{4}}$	M1
	$4(x+1)^{\frac{1}{4}}=t\ (+C)$	M1A1
	$x=15, t=0 \Rightarrow C=8$ so $4v^{\frac{1}{3}}=t+8$	M1
	$t=4v^{\frac{1}{3}}-8$	A1 (5)
	OR	
	$\frac{dv}{dt}=\frac{3}{4}v^{\frac{3}{4}}$	M1
	$3v^{\frac{1}{4}}=\frac{3}{4}t\ (+C)$	M1A1
	$t=0, v=8 \Rightarrow C=6$ so $3v^{\frac{1}{3}}=\frac{3}{4}t+6$	M1
	$t=4v^{\frac{1}{3}}-8$	A1 (5)
		(9)
	Notes for question 3	
3(a)		
M1	Set up a differential equation in v and x only M0 if acceleration is $\frac{dv}{dx}$ or $\frac{dv}{dt}$ M0 if there is no differential equation eg starting with $\frac{1}{2}v^2=\int\frac{3\sqrt{x+1}}{4}\,dx$	
M1	Clear attempt to separate variables and integrate acceleration in terms of v and x. At least one of the powers must increase by 1.	
A1	Correct integration, condone missing + C	
A1*	Given answer obtained from complete and correct working. Must include use of the boundary conditions and the initial differential equation. A0 if +C is not dealt with correctly eg If +C is only considered *after* the square root.	
3(b)		
M1	Set up a differential equation in x and t only. Using the given answer in (a).	
M1	Clear attempt to separate the variables and integrate in terms of x and t. At least one of the powers must increase by 1	
A1	Correct integration, condone missing + C	
M1	Use of boundary conditions in an integrated equation and use of (a) to form an equation in v and t. M0 if boundary conditions are not used.	
A1	Correct answer	
	OR	
M1	Set up a differential equation in v and t only	

Question Number	Scheme	Marks
3		
	Attempt to find final extension: $\sqrt{\left(\left(\frac{3a}{2}\right)^2+(2a)^2\right)}-a$	M1
	Method to find at least one expression for EPE	M1
	Two correct expressions for EPE (final and initial) $\frac{mg}{2a}\left(\frac{3a}{2}\right)^2, \frac{mg}{2a}\left(\frac{a}{2}\right)^2$	A1
	GPE Loss = $mg\times 2a$	B1
	Use of conservation of mechanical energy	M1
	$mg\times 2a+\frac{1}{2}mag+\frac{mg}{2a}\left(\frac{a}{2}\right)^2=\frac{1}{2}mV^2+\frac{mg}{2a}\left(\frac{3a}{2}\right)^2$ $\left(2mga+\frac{1}{2}mag+\frac{mga}{8}=\frac{1}{2}mV^2+\frac{9mga}{8}\right)$	A1
	$(V=)\ \sqrt{3ag}$	A1
		(7)
	Notes for question 3	
M1	Complete method to find the final extension (their $OB-a$). May see use of the 3,4,5 triangle or Pythagoras to find OB. May be implied by a correct final extension.	
M1	Method using EPE formula at least once. EPE must have the form $\frac{\lambda x^2}{k\,a}$ where λ is modulus of elasticity, x is their extension and k is a constant (condone $k=1$).	
A1	Two correct expressions for EPE	
B1	GPE term seen or implied	
M1	Use of the principle of conservation of mechanical energy. All required terms present and of the correct structure with no extras (2 EPE, 2KE, GPE). Condone sign errors. Note there are different rearrangements. For example, Initial = Final $mg\times 2a+\frac{1}{2}mag+\frac{mg}{2a}\left(\frac{a}{2}\right)^2=\frac{1}{2}mV^2+\frac{mg}{2a}\left(\frac{3a}{2}\right)^2$	

Question Number	Scheme	Marks
5(a)	Period $=12.25$ hours	B1
	$\frac{2\pi}{\omega}=12.25$	
	$\omega=\frac{2\pi}{12.25}$ oe eg $\omega=\frac{8\pi}{49}$	M1A1ft
	Amplitude $=5$ m	B1
	$v^2=\omega^2\left(a^2-x^2\right)$	
	$x=4 \Rightarrow v^2=\left(\frac{2\pi}{12.25}\right)^2\left(5^2-4^2\right)$	M1A1
	$v=\left(\frac{2\pi}{12.25}\right)\times 3=1.53873...=1.5$ m h^{-1} (1.5 or better or $\frac{24\pi}{49}$)	A1cao (7)
(b)	$x=4 \quad 4=5\cos\omega t$	
	$t=\frac{12.25}{2\pi}\cos^{-1}0.8$ $(=1.2546...)$	M1A1
	Time ladder in the water $=12.25-2\times\frac{12.25}{2\pi}\cos^{-1}0.8$	DM1
	$=9.7407...=9.7$ hours (9.7 or better) (9hr 44 min or 35 066 875 seconds)	A1 cao (4) [11]

(a)B1 Correct period for the SHM (Allow 44100 s)

M1 Using period $=\frac{2\pi}{\omega}$ to obtain an expression for ω or any other complete method

A1ft A correct expression for ω follow through their period, inc with units changed.

B1 Correct amplitude seen explicitly or used

M1 Using $v^2=\omega^2\left(a^2-x^2\right)$ with their ω, amplitude and $x=\left(\text{their amp}-1\right)$

A1 Correct expression for v^2

A1cao Correct value of v, 2 sf or better Units must be m h^{-1}.

(b)M1 Using $x=a\cos\omega t$ or $a\sin\omega t$ with $x=\pm\left(\text{their amp}-1\right)$, their ω and amplitude

A1 Finding a correct time, either from the end point to reaching the bottom step or from the bottom step to the centre of the oscillation. (in hours or seconds)

DM1 Using their time to obtain an expression for the required time

A1cao Time = 9.7 hrs or better

Sine used: Time $=2\times\frac{\text{period}}{4}+2\times\text{time found}$

June 2018
WME03 M3
Mark Scheme

Question Number	Scheme	Marks
1.	$T = \dfrac{29.4(0.5-l)}{l}$ $1.5g = \dfrac{29.4(0.5-l)}{l}$ $1.5 = \dfrac{1.5-3l}{l}$ $3l = 1 \quad l = 0.33$ or $0.333\,\text{m}$ must be 2 or 3 sf	M1A1 dM1A1 [4]

M1	Attempt Hooke's Law with one unknown (natural length or extension but not both) Extension = 0.5 scores M0. T or 1.5 for this mark
A1	Fully correct vertical equation
dM1	Solve their equation depends on the first M mark
A1	Correct natural length Ignore $l = \dfrac{1}{3}$ if 0.33 or 0.33 also seen
NB:	If extension is used as the unknown and work not completed to give the natural length, first M1A1 only available.