Leave blank

6. A smooth sphere, with centre O and radius a, is fixed with its lowest point A on a horizontal floor. A particle P is placed on the surface of the sphere at the point B, where B is vertically above A. The particle is projected horizontally from B with speed $\sqrt{\frac{ag}{5}}$ and moves along the surface of the sphere. When OP makes an angle θ with the upward vertical, and P is still in contact with the sphere, the speed of P is v.

(a) Show that $v^2 = \frac{ag}{5}(11 - 10\cos\theta)$.

(4)

The particle leaves the surface of the sphere at the point C.

Find

(b) the speed of P at C in terms of a and g,

(6)

(c) the size of the angle between the floor and the direction of motion of P at the instant immediately before P hits the floor.

(5)

2. A light elastic string AB has natural length $3a$ and modulus of elasticity $\frac{20mg}{7}$

When the string is unstretched, two particles, each of mass m, are attached to the string, one at P, where $AP = a$ and the other at Q, where $AQ = 2a$.

The end A of the string is then attached to a point X on a horizontal ceiling and the end B is attached to another point Y on the ceiling, where $XY > 3a$.

The particles hang at rest in equilibrium and the two portions of the string, XP and YQ, both make an angle θ with the horizontal, as shown in Figure 1.

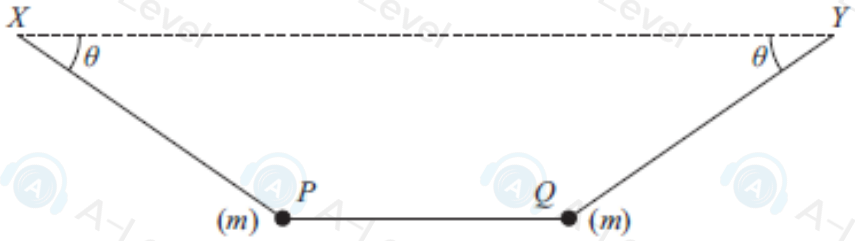

Figure 1

Given that $\tan\theta = \frac{3}{4}$, find XY in terms of a.

(10)

3.

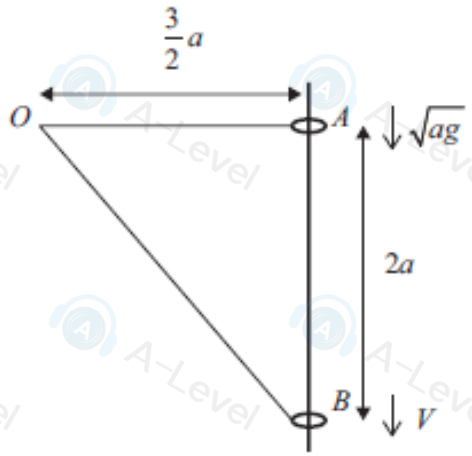

Figure 2

A light elastic string has natural length a and modulus of elasticity mg.
One end of the elastic string is attached to a fixed point O.
The other end is attached to a small smooth ring of mass m.

The ring is threaded on a fixed smooth vertical pole which is a distance $\frac{3}{2}a$ from O.

Initially, the ring is held at the point A on the pole with the elastic string horizontal.

The ring is then projected vertically downwards with speed $\sqrt{ag}$ and reaches the point B, where $AB = 2a$, with speed V, as shown in Figure 2.
Air resistance is assumed to be negligible.

Find V in terms of a and g.

(7)

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5. A particle P of mass 0.4 kg moves on the positive x-axis under the action of a single force.

The force is directed towards the origin O and has magnitude $\frac{k}{x^2}$ newtons, where $OP = x$ metres and k is a constant. Initially P is moving away from O. At $x = 2$ the speed of P is 5 m s^{-1} and at $x = 5$ the speed of P is 2 m s^{-1}.

(a) Find the value of k.

(8)

The particle first comes to instantaneous rest at the point A.

(b) Find the value of x at A.

(4)