4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 2^x$, show that the equation

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

can be written in the form

$$4p^2 - 33p + 8 = 0$$

(3)

(b) Hence solve

$$2 \times 4^x - 2^{x+3} = 17 \times 2^{x-1} - 4$$

(3)

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5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)

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8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

7. The curve C has equation $y = f(x)$ where

$$f(x) = 2x^3 - kx^2 + 14x + 24$$

and k is a constant.

(a) Find, in simplest form,

(i) $f'(x)$

(ii) $f''(x)$

(3)

The curve with equation $y = f'(x)$ intersects the curve with equation $y = f''(x)$ at the points A and B.

Given that the x coordinate of A is 5

(b) find the value of k.

(2)

(c) Hence find the coordinates of B.

(3)

5.

$$y = \frac{1}{2}x^4 - 3 + \frac{10}{x^2} \qquad x \neq 0$$

(a) Find $\int y \, dx$ writing the answer in simplest form.

(3)

(b) (i) Find $\frac{dy}{dx}$ writing the answer in simplest form.

(3)

(ii) Hence find the exact solutions of the equation $\frac{dy}{dx} = 3$

(*Solutions relying on calculator technology are not acceptable.*)

(4)

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2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3\left(9^{3z}\right) + 26\left(9^{2z}\right) - 9\left(9^{z}\right) = 0$

(5)

7.

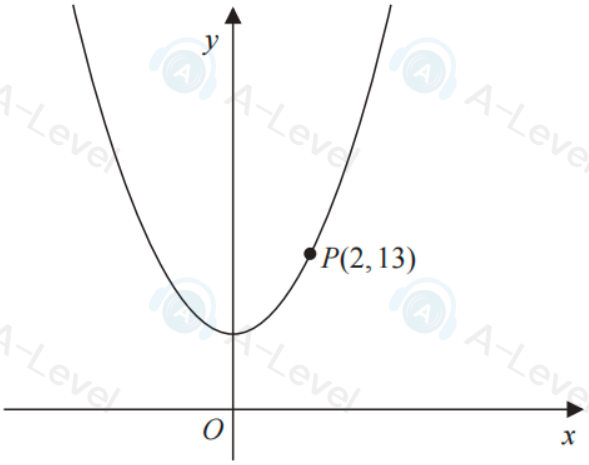

Figure 4

Figure 4 shows part of the curve with equation $y = 2x^2 + 5$

The point $P(2,13)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Give your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

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1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form.

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

(2)

8.

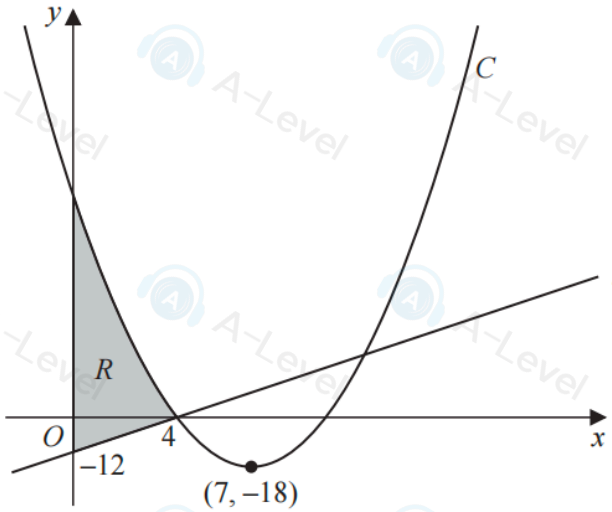

Figure 2

Figure 2 shows a sketch of the straight line l and the curve C.

Given that l cuts the y-axis at -12 and cuts the x-axis at 4, as shown in Figure 2,

(a) find an equation for l, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

Given that C

- has equation $y = f(x)$ where $f(x)$ is a quadratic expression
- has a minimum point at $(7, -18)$
- cuts the x-axis at 4 and at k, where k is a constant

(b) deduce the value of k,

(1)

(c) find $f(x)$.

(3)

The region R is shown shaded in Figure 2.

(d) Use inequalities to define R.

(2)

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7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

(b) Solve the equation

$$f''(x) = 6$$

(5)

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11.

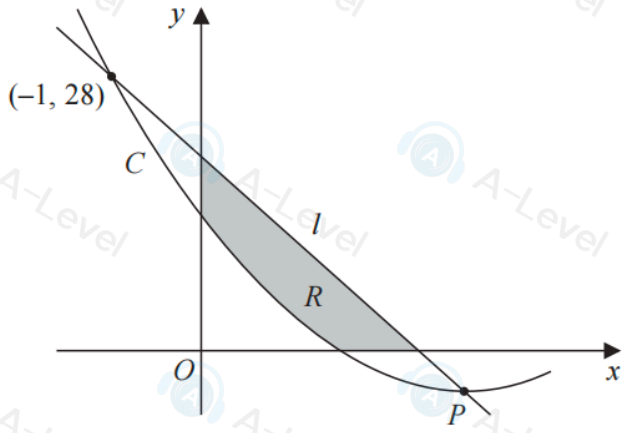

Figure 5

Figure 5 shows part of the curve C with equation $y = f(x)$ where

$$f(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

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3.

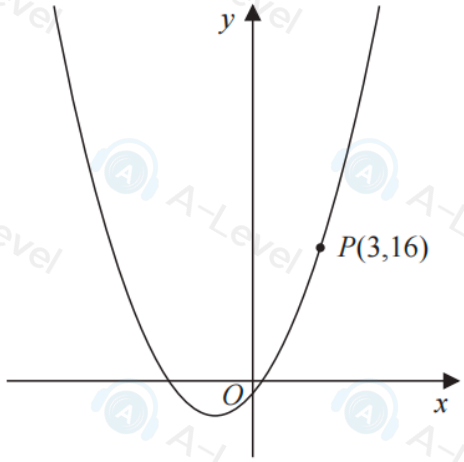

Figure 1

Figure 1 shows part of the curve with equation $y = x^2 + 3x - 2$

The point $P(3,16)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $3 + h$ also lies on the curve.

(b) Find, in terms of h, the gradient of the line PQ. Write your answer in simplest form.

(3)

(c) Explain briefly the relationship between the answer to (b) and the answer to (a).

(1)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Fully factorise

$$9x^3 - 10x^2 + x$$

(2)

(b) Hence solve

$$9 \times 27^y - 10 \times 9^y + 3^y = 0$$

(3)

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