4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given that

$$y = a^x \qquad \text{where } a \text{ is a positive constant}$$

express, in simplest form, in terms of y and a

(a) a^{3x+1}

(1)

(b) $\dfrac{5}{\left(3a^{1-x}\right)^{-2}}$

(3)

(ii) (a) Use the substitution $p = 9^t$ to show that the equation

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

can be rewritten as

$$27p^2 - 82p + 3 = 0$$

(2)

(b) Hence solve

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

(3)

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2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3\left(9^{3z}\right) + 26\left(9^{2z}\right) - 9\left(9^{z}\right) = 0$

(5)

5. (a) Find, using algebra, all real solutions of

$$2x^3 + 3x^2 - 35x = 0$$

(3)

(b) Hence find all real solutions of

$$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$$

(4)

2. Given $y = 3^x$, express each of the following in terms of y. Write each expression in its simplest form.

(a) 3^{3x}

(1)

(b) $\dfrac{1}{3^{x-2}}$

(2)

(c) $\dfrac{81}{9^{2-3x}}$

(2)

2. Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form kx^n where k and n are simplified constants.

(a) $a^{\frac{1}{2}}$

(1)

(b) $\dfrac{16}{b^3}$

(1)

(c) $\left(\dfrac{ab}{2}\right)^{-\frac{4}{3}}$

(2)

1. Given that

$$p = \frac{1}{16}x^4 \qquad q = \frac{40}{x^3}$$

express each of the following in the form kx^n where k and n are fully simplified constants.

(a) $p^{\frac{1}{2}}$

(1)

(b) $(pq)^{-1}$

(2)

(c) pq^2

(2)

4.

(i) Using the laws of indices, solve

$$2^{4k-3} = \frac{8^{1-k}}{4\sqrt{2}}$$

(3)

(ii) Solve the equation

$$\frac{x\sqrt{3} + 2}{\sqrt{3} - 1} = x\sqrt{3} - 4$$

giving the answer in the form $a + b\sqrt{3}$, where a and b are rational numbers.

(4)

DO NOT WRITE IN THIS AREA