6. The curve C has equation $y = \frac{4}{x} + k$, where k is a positive constant.

(a) Sketch a graph of C, stating the equation of the horizontal asymptote and the coordinates of the point of intersection with the x-axis.

(3)

The line with equation $y = 10 - 2x$ is a tangent to C.

(b) Find the possible values for k.

(5)

1. Find

$$\int \left(\frac{1}{2}x^3 + \frac{3}{\sqrt{x}} - 4 \right) dx$$

writing your answer in simplest form.

(4)

10. The curve C has equation $y = f(x)$.

Given that

- $f'(x) = \frac{k\sqrt{x}(x-3)}{5}$ where k is a constant
- the point P with x coordinate 4 lies on C
- the equation of the normal to C at P is $y = -\frac{5}{4}x + 2$

(a) find an equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants,

(3)

(b) find the value of k,

(2)

(c) find $f(x)$, writing the answer in simplest form.

(5)

7 The curve $y = f(x)$ is such that $f'(x) = \frac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. [4]

5. $$y = \frac{1}{2}x^4 - 3 + \frac{10}{x^2} \qquad x \neq 0$$

(a) Find $\int y \, \mathrm{d}x$ writing the answer in simplest form.

(3)

(b) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ writing the answer in simplest form.

(3)

(ii) Hence find the exact solutions of the equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$

(*Solutions relying on calculator technology are not acceptable.*)

(4)

8. A curve C has equation $y = \mathrm{f}(x)$.

The point P with x coordinate 3 lies on C

Given

- $\mathrm{f}'(x) = 4x^2 + kx + 3$ where k is a constant
- the normal to C at P has equation $y = -\frac{1}{24}x + 5$

(a) show that $k = -5$

(3)

(b) Hence find $\mathrm{f}(x)$.

(4)

9.

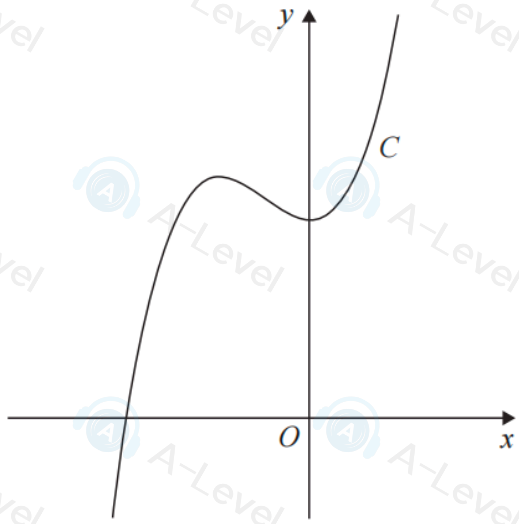

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $f(x) \geqslant 0$

(1)

(b) Find $f'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = f(x)$ is transformed to the curve with equation,

(i) $y = f(x - 3)$

(ii) $y = 4f(x)$

(2)

blank

1. Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx$$

giving each term in simplest form.

(5)

Leave blank

11. A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}''(x) = \frac{6}{\sqrt{x^3}} + x \qquad x > 0$$

The point $P(4, -50)$ lies on the curve.

Given that $\mathrm{f}'(x) = -4$ at P,

(a) find the equation of the normal at P, writing your answer in the form $y = mx + c$, where m and c are constants,

(3)

(b) find $\mathrm{f}(x)$.

(8)

6.

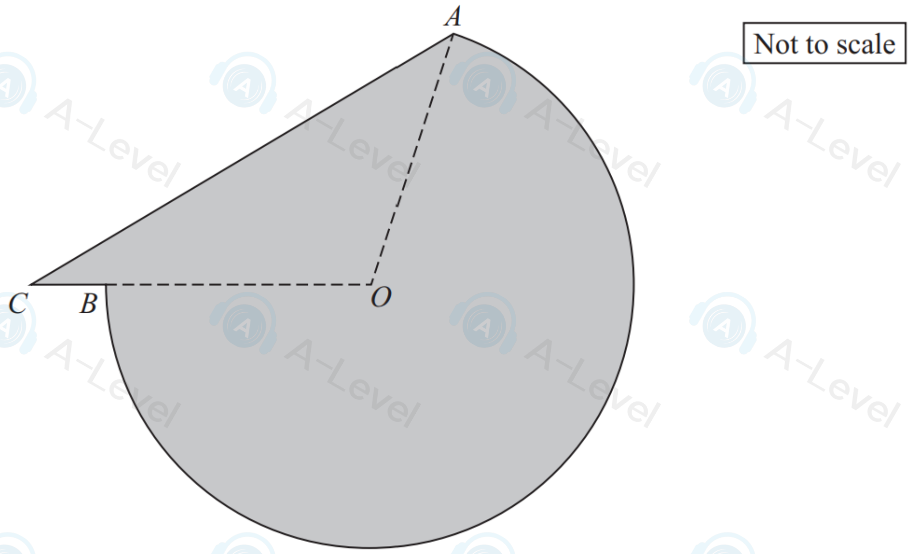

Figure 2

The shaded area in Figure 2 shows the plan view of a helicopter landing pad.

The area consists of the major sector AOB of a circle centre O joined to a triangle AOC.

Given that

- $AO = OB = 15\,\mathrm{m}$
- $BC = 2\,\mathrm{m}$
- CBO is a straight line
- angle $ACO = 0.6$ radians

(a) show that angle COA is 1.847 radians to 3 decimal places.

(3)

(b) Find the total area of the helicopter landing pad.
Give your answer in m^2 to 3 significant figures.

(3)

(c) Find the perimeter of the helicopter landing pad.
Give your answer in metres to 3 significant figures.

(3)

8. The curve C has equation

$$y = (x-2)(x-4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

blank

6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

(2)

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

(3)

(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Simplify fully

$$\frac{3y^3(2x^4)^3}{4x^2y^4}$$

(3)

(ii) Find the exact value of a such that

$$\frac{16}{\sqrt{3}+1} = a\sqrt{27} + 4$$

Write your answer in the form $p\sqrt{3} + q$ where p and q are fully simplified rational constants.

(4)