

6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$

$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x+p) + q$, where p and q are constants. [4]

6. The curve C has equation $y = f(x)$ where $x > 0$

Given that

- $f'(x) = \frac{(x+3)^2}{x\sqrt{x}}$
- the point $P(4, 20)$ lies on C

(a) (i) find the value of the gradient at P

(ii) Hence find the equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found. (4)

(b) Find $f(x)$, simplifying your answer. (7)

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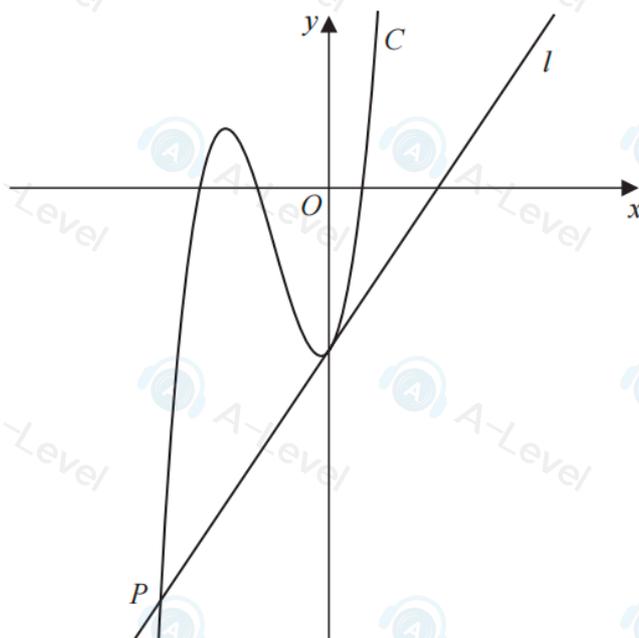


Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = (3x + 20)(x + 6)(2x - 3)$$

(a) Use the given information to state the values of x for which

$$f(x) > 0$$

(2)

(b) Expand $(3x + 20)(x + 6)(2x - 3)$, writing your answer as a polynomial in simplest form.

(3)

The straight line l is the tangent to C at the point where C cuts the y -axis.

Given that l cuts C at the point P , as shown in Figure 4,

(c) find, using algebra, the x coordinate of P

(Solutions based on calculator technology are not acceptable.)

(5)

5.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \quad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k .

(4)

blank

7.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \quad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

(b) Solve the equation

$$f''(x) = 6$$

(5)

5.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$

- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $f(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y -axis.

(4)

10.

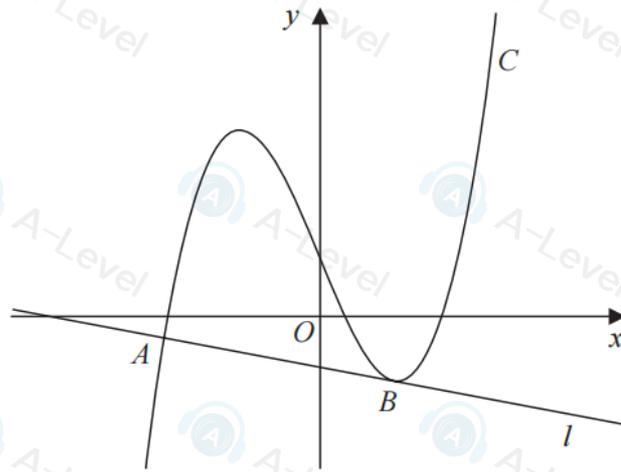


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The line l , shown in Figure 5, is the normal to C at the point A with x coordinate $-\frac{7}{2}$

Given that l is also a tangent to C at the point B ,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B , justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k .

(4)

10.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{3}x^3 - 25x - \frac{56}{x} + \frac{194}{3} \quad x > 0$$

The point P , which lies on C , has coordinates $(2, -8)$

(a) Show that an equation of the tangent to C at P is

$$y = -3x - 2 \quad (5)$$

The point Q also lies on C .

Given that the tangent to C at Q is parallel to the tangent to C at P ,

(b) find, using algebra and showing your working, the exact x coordinate of Q .

(5)

2. The curve C has equation

$$y = 2x^{\frac{5}{2}} - 4x + 3$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

The point P lies on C .

Given that

- the x coordinate of P is 2^k where k is a constant
- the gradient of C at the point P is 16

(b) find the value of k .

(3)

10.

In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**The curve C has equation

$$y = \frac{2}{3}x^3 - 25x - \frac{56}{x} + \frac{194}{3} \quad x > 0$$

The point P , which lies on C , has coordinates $(2, -8)$ (a) Show that an equation of the tangent to C at P is

$$y = -3x - 2 \quad (5)$$

The point Q also lies on C .Given that the tangent to C at Q is parallel to the tangent to C at P ,(b) find, using algebra and showing your working, the exact x coordinate of Q .

(5)

2 (a) Find the first three terms, in ascending powers of x , in the expansion of $(1 + ax)^6$.

[1]

8.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$ Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(5)

5.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \quad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k .

(4)