8. The curve C_1 has equation

$$y = x(4 - x^2)$$

(a) Sketch the graph of C_1 showing the coordinates of any points of intersection with the coordinate axes.

(3)

The curve C_2 has equation $y = \dfrac{A}{x}$ where A is a constant.

(b) Show that the x coordinates of the points of intersection of C_1 and C_2 satisfy the equation

$$x^4 - 4x^2 + A = 0$$

(1)

(c) Hence find the range of possible values of A for which C_1 meets C_2 at 4 distinct points.

(3)

DO NOT WRITE IN THIS AREA

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The line l_1 has equation

$$x - 2y + 25 = 0$$

The line l_2 passes through the origin and is perpendicular to l_1

(a) Find an equation for l_2

(2)

The lines l_1 and l_2 intersect at the point P.

(b) Use algebra to find the coordinates of P.

(3)

(c) Hence find the shortest distance from l_1 to the origin.

Write your answer as a fully simplified surd.

(2)

DO NOT WRITE IN THIS AREA

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

9.

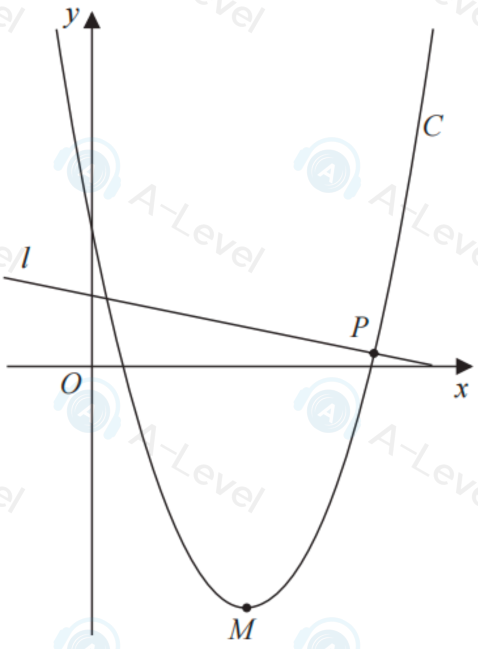

Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = \frac{1}{2}x^2 - 10x + 22$$

(a) Write $\frac{1}{2}x^2 - 10x + 22$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point M is the minimum turning point of C, as shown in Figure 3.

(b) Deduce the coordinates of M

(2)

The line l is the normal to C at the point P, as shown in Figure 3.

Given that l has equation $y = k - \frac{1}{8}x$, where k is a constant,

(c) (i) find the coordinates of P

(ii) find the value of k

(6)

Question 9 continued

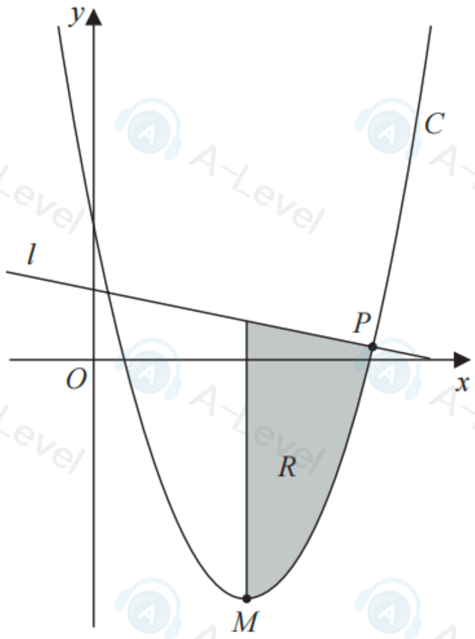

Figure 4

Figure 4 is a copy of Figure 3. The finite region R, shown shaded in Figure 4, is bounded by l, C and the line through M parallel to the y-axis.

(d) Identify the inequalities that define R.

(3)

1. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the inequality

$$4x^2 - 3x + 7 \geqslant 4x + 9$$

(4)

11.

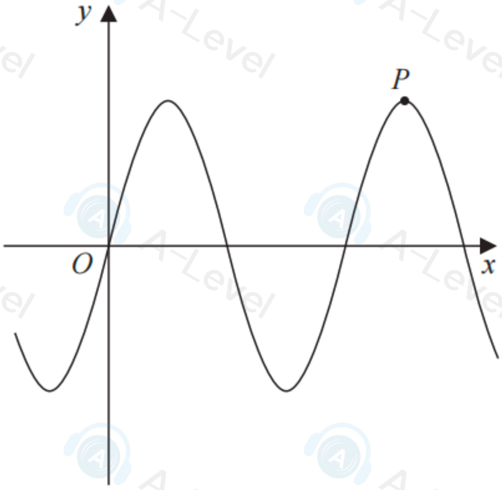

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P.

(2)

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

1. The line l_1 passes through the point $A(-5, 20)$ and the point $B(3, -4)$.

(a) Find an equation for l_1 giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

The line l_2 is perpendicular to l_1 and passes through the midpoint of AB

(b) Find an equation for l_2 giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

(3)