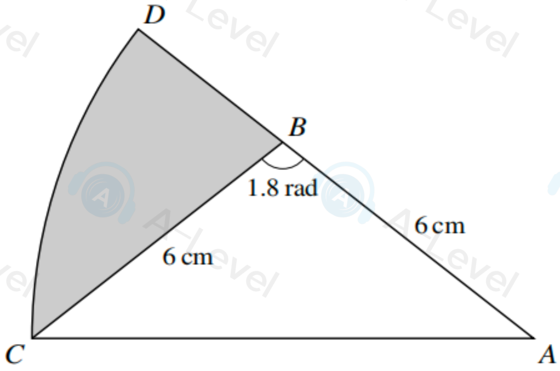
9



The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

7.

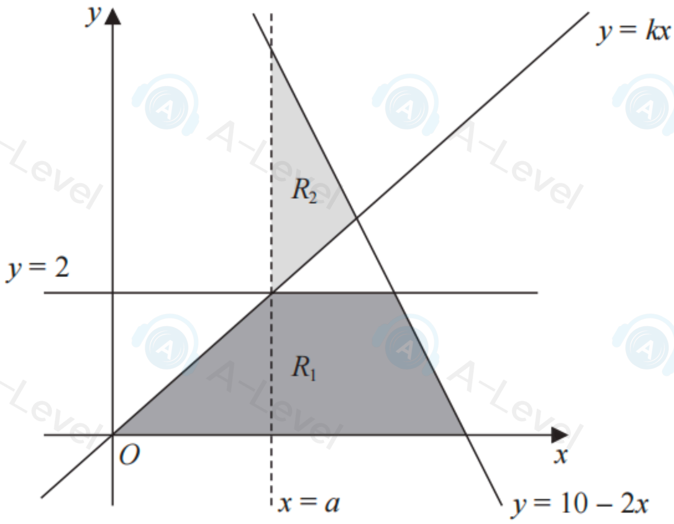


Figure 2

The region R_1, shown shaded in Figure 2, is defined by the inequalities

$$0 \leqslant y \leqslant 2 \qquad y \leqslant 10 - 2x \qquad y \leqslant kx$$

where k is a constant.

The line $x = a$, where a is a constant, passes through the intersection of the lines $y = 2$ and $y = kx$

Given that the area of R_1 is $\frac{27}{4}$ square units,

(a) find

(i) the value of a

(ii) the value of k

(4)

(b) Define the region R_2, also shown shaded in Figure 2, using inequalities.

(2)

blank

6. (a) Sketch the curve with equation

$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

(2)

(b) On a separate diagram, sketch the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

(3)

(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)

DO NOT WRITE IN THIS AREA

1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$

(3)

(b) $\frac{d^2y}{dx^2}$

(2)

DO NOT WRITE IN THIS AREA

9. The curve C_1 has equation $y = f(x)$.

Given that

- $f(x)$ is a quadratic expression
- C_1 has a maximum turning point at (2, 20)
- C_1 passes through the origin

(a) sketch a graph of C_1 showing the coordinates of any points where C_1 cuts the coordinate axes,

(2)

(b) find an expression for $f(x)$.

(3)

The curve C_2 has equation $y = x(x^2 - 4)$

Curve C_1 and C_2 meet at the origin, and at the points P and Q

Given that the x coordinate of the point P is negative,

(c) using algebra and showing all stages of your working, find the coordinates of P

(5)

7.

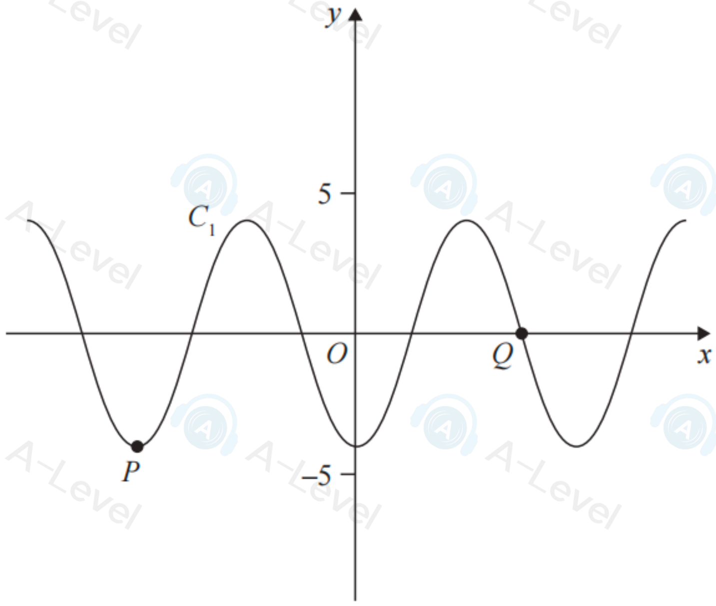


Figure 3

Figure 3 shows a plot of part of the curve C_1 with equation

$$y = -4\cos x$$

where x is measured in radians.

Points P and Q lie on the curve and are shown in Figure 3.

(a) State

(i) the coordinates of P

(ii) the coordinates of Q

(3)

The curve C_2 has equation $y = -4\cos x + k$ where x is measured in radians and k is a constant.

Given that C_2 has a maximum y value of 11

(b) (i) state the value of k

(ii) state the coordinates of the minimum point on C_2 with the smallest positive x coordinate.

(3)

On the opposite page there is a copy of Figure 3 labelled Diagram 1.

(c) Using Diagram 1, state the number of solutions of the equation

$$-4\cos x = 5 - \frac{10}{\pi}x$$

giving a reason for your answer.

(2)

5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

10.

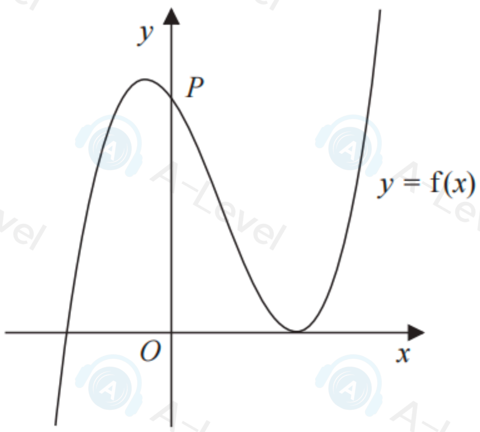


Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x + 5)(x - 3)^2$$

(a) Deduce the values of x for which $f(x) \leqslant 0$

(2)

The curve crosses the y-axis at the point P, as shown.

(b) Expand $f(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

(c) Hence, or otherwise, find

(i) the coordinates of P,

(ii) the gradient of the curve at P.

(2)

The curve with equation $y = f(x)$ is translated two units in the positive x direction to a curve with equation $y = g(x)$.

(d) (i) Find $g(x)$, giving your answer in a simplified factorised form.

(ii) Hence state the y intercept of the curve with equation $y = g(x)$.

(3)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The line l_1 has equation

$$x - 2y + 25 = 0$$

The line l_2 passes through the origin and is perpendicular to l_1

(a) Find an equation for l_2

(2)

The lines l_1 and l_2 intersect at the point P.

(b) Use algebra to find the coordinates of P.

(3)

(c) Hence find the shortest distance from l_1 to the origin.

Write your answer as a fully simplified surd.

(2)