7.

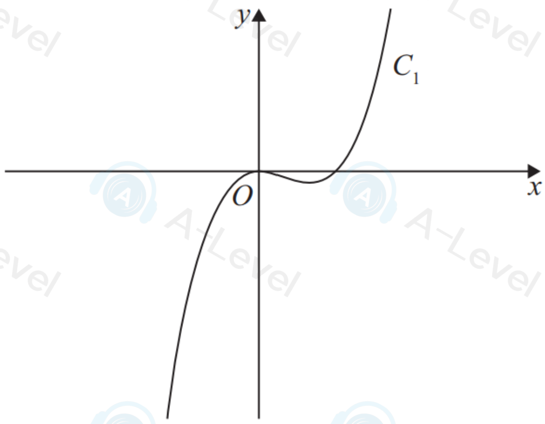

Figure 3

Figure 3 shows a sketch of part of the curve C_1

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a cubic function
- touches the x-axis at the origin and cuts the x-axis at $x = 4$
- passes through the point $(10, 120)$

(a) find $f(x)$

(3)

The curve C_2 has equation $y = 1.2x(8 - x)$

On the following page there is a copy of Figure 3 called Diagram 1.

(b) On Diagram 1 sketch a graph of the curve C_2

(2)

(c) Use algebra to find the coordinates of the points where C_1 and C_2 intersect. Show each stage of your working.

(5)

3. **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(a) Find $\int f(x)\,dx$

(4)

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)

10. **In this question you must show all stages of your working.**

The curve C has equation $y = f(x), x > 0$

Given that

- the point $P\left(2, 8\sqrt{2}\right)$ lies on C
- $f'(x) = 4\sqrt{x^3} + \dfrac{k}{x^2}$ where k is a constant
- $f''(x) = 0$ at P

(a) find the exact value of k, **(4)**

(b) find $f(x)$, giving your answer in simplest form. **(4)**

11.

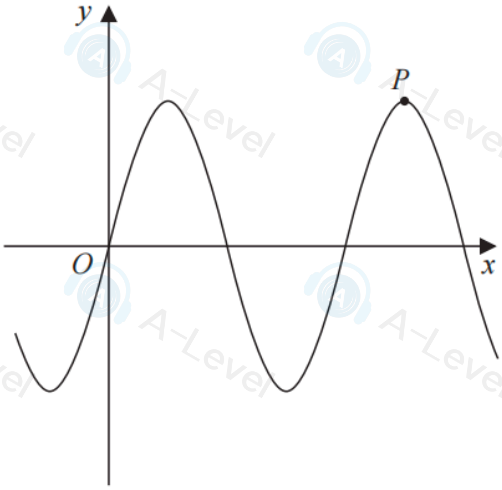

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 12\sin x$$

where x is measured in radians.

The point P shown in Figure 4 is a maximum point on C_1

(a) Find the coordinates of P. **(2)**

The curve C_2 has equation

$$y = 12\sin x + k$$

where k is a constant.

Given that the **maximum** value of y on C_2 is 3

(b) find the coordinates of the **minimum** point on C_2 which has the **smallest** positive x coordinate.

(2)

The curve C_3 has equation

$$y = 12\sin(x + B)$$

where B is a positive constant.

Given that $\left(\frac{\pi}{4}, A\right)$, where A is a constant, is the **minimum** point on C_3 which has the **smallest** positive x coordinate,

(c) find

(i) the value of A,

(ii) the smallest possible value of B.

(2)

7.

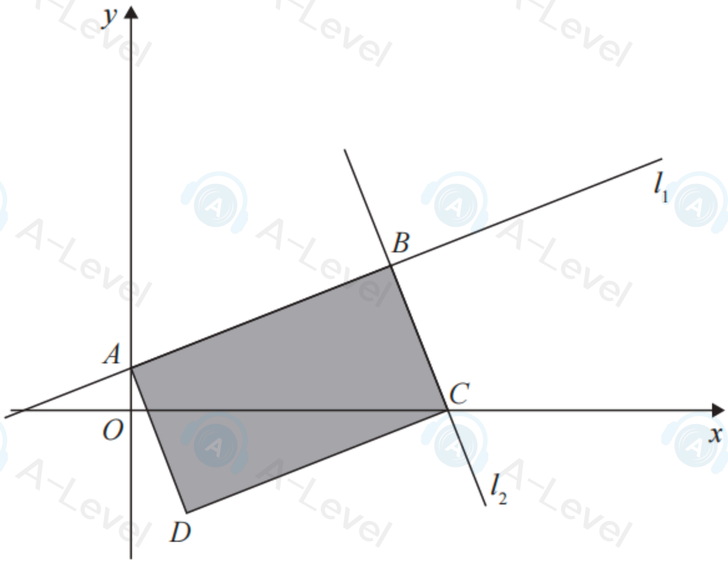

Figure 2

The straight line l_1 shown in Figure 2 has equation $5y = 2x + 10$

The points A and B lie on l_1 such that

- point A lies on the y-axis
- point B has x coordinate 10

(a) Find the distance AB writing your answer as a fully simplified surd.

(3)

The straight line l_2 also shown in Figure 2

- passes through B
- is perpendicular to l_1

(b) Find an equation for l_2 writing your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Line l_2 crosses the x-axis at the point C.

Point D is such that the points A, B, C and D form the vertices of a rectangle, shown shaded in Figure 2.

(c) Find the area of rectangle $ABCD$.

(3)

1. Given that

$$p = \frac{1}{16}x^4 \qquad q = \frac{40}{x^3}$$

express each of the following in the form kx^n where k and n are fully simplified constants.

(a) $p^{\frac{1}{2}}$ **(1)**

(b) $(pq)^{-1}$ **(2)**

(c) pq^2 **(2)**

9.

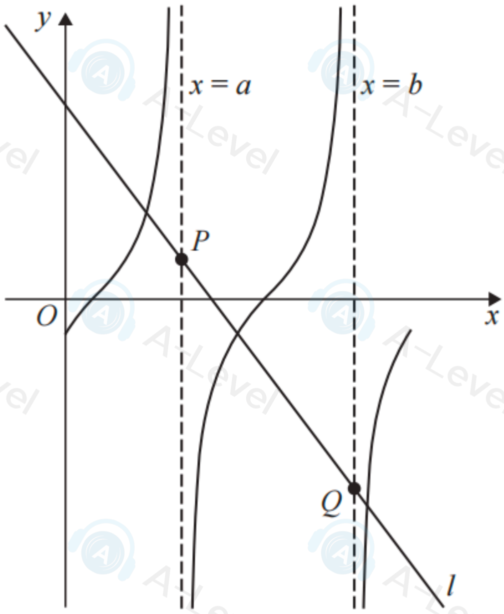

Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan\left(x - \frac{\pi}{6}\right)$ for $0 \leqslant x \leqslant 2\pi$
- part of the straight line l with equation $y = \pi - x$

(a) State the number of solutions of the equation

(i) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 2\pi$

(ii) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 100\pi$

(iii) $\tan\left(x - \frac{\pi}{6}\right) = \pi + x$ in the interval $0 \leqslant x \leqslant 2\pi$ **(3)**

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The line with equation $x = a$, shown in Figure 3, is the asymptote to the curve with the smallest positive x coordinate.

(b) State the value of a

(1)

The line with equation $x = b$, also shown in Figure 3, is the asymptote to the curve with the second smallest positive x coordinate.

The line l meets $x = a$ at point P and meets $x = b$ at point Q as shown in Figure 3.

(c) Find the midpoint of the line segment PQ.

(4)

8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

5.

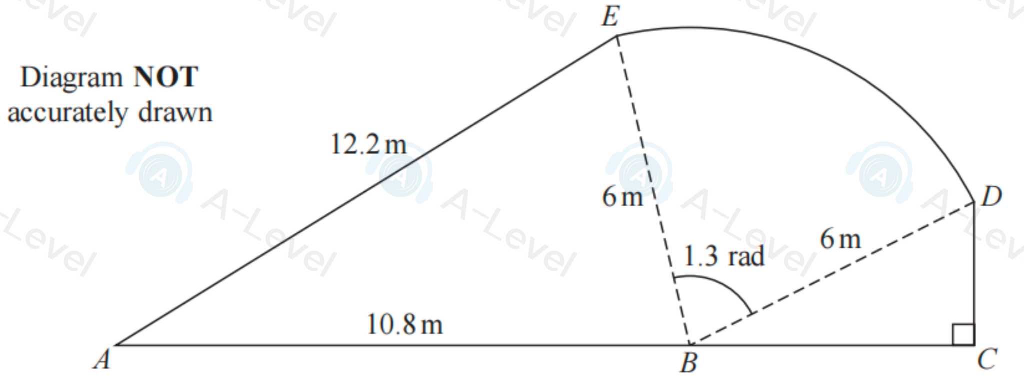

Figure 2

Figure 2 shows the plan view of a garden.

The shape of the garden *ABCDEA* consists of a triangle *ABE* and a right-angled triangle *BCD* joined to a sector *BDE* of a circle with radius 6 m and centre *B*.

The points *A*, *B* and *C* lie on a straight line with $AB = 10.8$ m

Angle $BCD = \frac{\pi}{2}$ radians, angle $EBD = 1.3$ radians and $AE = 12.2$ m

(a) Find the area of the sector *BDE*, giving your answer in m^2

(2)

(b) Find the size of angle *ABE*, giving your answer in radians to 2 decimal places.

(2)

(c) Find the area of the garden, giving your answer in m^2 to 3 significant figures.

(3)