9. The curve C_1 has equation $y = f(x)$.

Given that

- $f(x)$ is a quadratic expression
- C_1 has a maximum turning point at (2, 20)
- C_1 passes through the origin

(a) sketch a graph of C_1 showing the coordinates of any points where C_1 cuts the coordinate axes,

(2)

(b) find an expression for $f(x)$.

(3)

The curve C_2 has equation $y = x(x^2 - 4)$

Curve C_1 and C_2 meet at the origin, and at the points P and Q

Given that the x coordinate of the point P is negative,

(c) using algebra and showing all stages of your working, find the coordinates of P

(5)

5. **In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.**

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)

4.

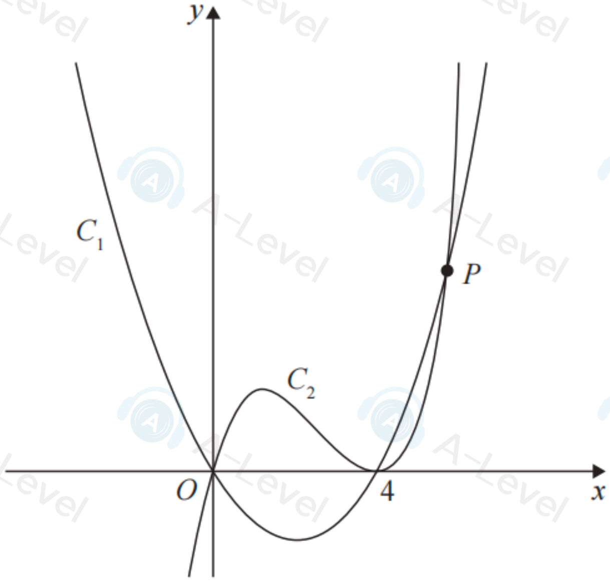

Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2

Given that C_1

- has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic function
- cuts the x-axis at the origin and at $x = 4$
- has a minimum turning point at $(2, -4.8)$

(a) find $\mathrm{f}(x)$

(3)

Given that C_2

- has equation $y = \mathrm{g}(x)$ where $\mathrm{g}(x)$ is a cubic function
- cuts the x-axis at the origin and meets the x-axis at $x = 4$
- passes through the point $(6, 7.2)$

(b) find $\mathrm{g}(x)$

(3)

The curves C_1 and C_2 meet in the first quadrant at the point P, shown in Figure 1.

(c) Use algebra to find the coordinates of P.

(4)

3.

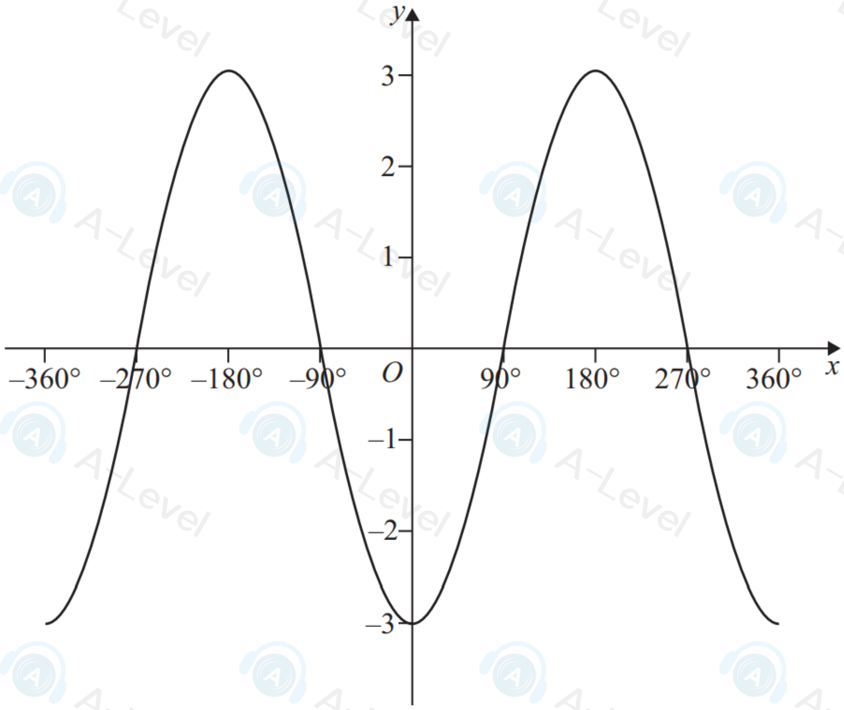

Figure 2

Figure 2 shows part of the graph of the trigonometric function with equation $y = \mathrm{f}(x)$, where x is measured in degrees.

(a) Write down an expression for $\mathrm{f}(x)$.

(2)

(b) State the number of solutions of the equation

(i) $\mathrm{f}(x) = 2$ in the interval $-720° \leqslant x \leqslant 720°$

(ii) $\mathrm{f}(x) = -3$ in the interval $-720° \leqslant x \leqslant 720°$

(2)

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5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

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7.

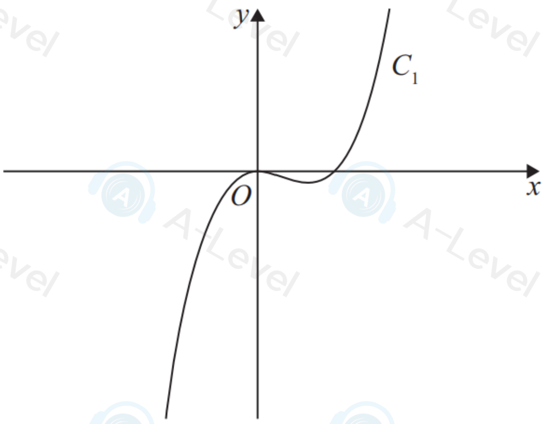

Figure 3

Figure 3 shows a sketch of part of the curve C_1

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a cubic function
- touches the x-axis at the origin and cuts the x-axis at $x = 4$
- passes through the point $(10, 120)$

(a) find $f(x)$

(3)

The curve C_2 has equation $y = 1.2x(8 - x)$

On the following page there is a copy of Figure 3 called Diagram 1.

(b) On Diagram 1 sketch a graph of the curve C_2

(2)

(c) Use algebra to find the coordinates of the points where C_1 and C_2 intersect. Show each stage of your working.

(5)

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6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$y = x^3 + 96\sqrt{x} + 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

(3)

(b) Find the solution of the equation

$$\frac{d^2y}{dx^2} = 0$$

writing the answer in the form 2^k where k is a constant.

(3)

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int\left(\frac{2}{3}x^3 - \frac{1}{2x^3} + 5\right)dx$$

simplifying your answer.

(4)

8.

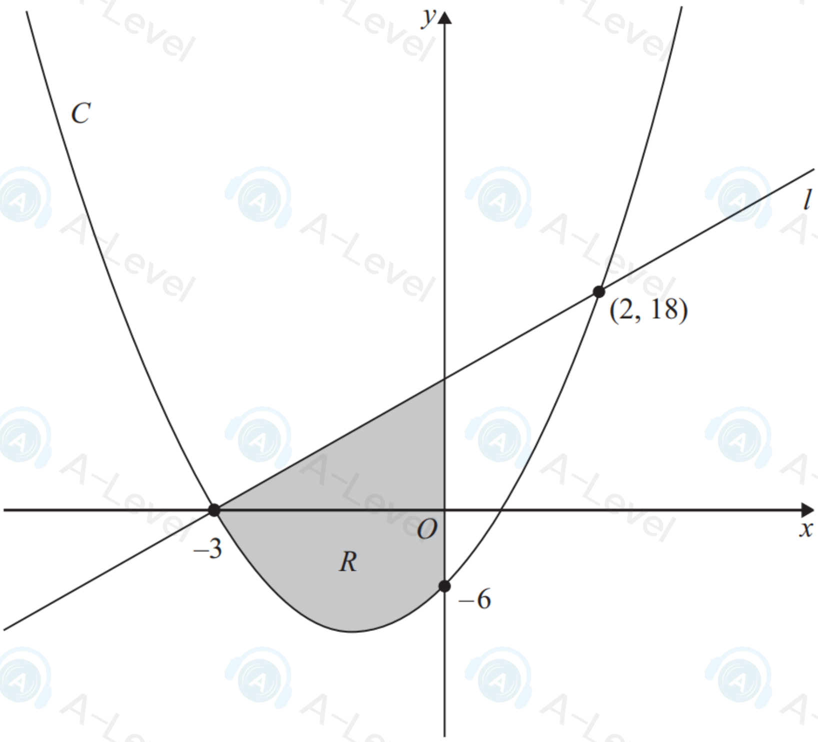

Figure 3

Figure 3 shows a sketch of a line l and a quadratic curve C.

Given that l passes through $(-3, 0)$ and $(2, 18)$

(a) find an equation for l in the form $y = mx + c$ where m and c are constants.

(3)

Given that

- C and l intersect at the points $(-3, 0)$ and $(2, 18)$
- C crosses the y-axis at $(0, -6)$

(b) find an equation for C.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Use inequalities to define R.

(2)

9.

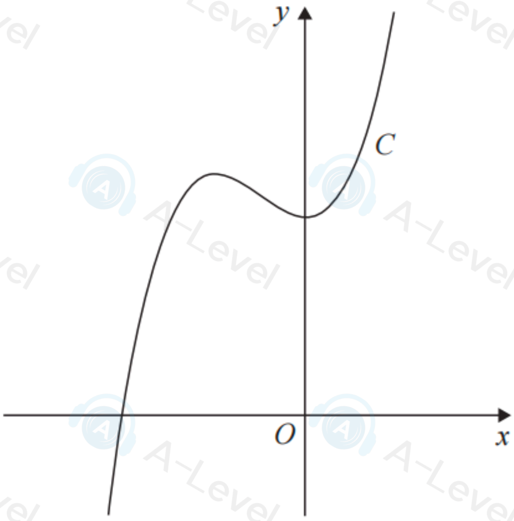

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

2. The curve C has equation

$$y = 2x^{\frac{5}{2}} - 4x + 3$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ writing your answer in simplest form.

(2)

The point P lies on C.

Given that

- the x coordinate of P is 2^k where k is a constant
- the gradient of C at the point P is 16

(b) find the value of k.

(3)