9. The curve C_1 has equation $y = f(x)$.

Given that

- $f(x)$ is a quadratic expression
- C_1 has a maximum turning point at (2, 20)
- C_1 passes through the origin

(a) sketch a graph of C_1 showing the coordinates of any points where C_1 cuts the coordinate axes,

(2)

(b) find an expression for $f(x)$.

(3)

The curve C_2 has equation $y = x(x^2 - 4)$

Curve C_1 and C_2 meet at the origin, and at the points P and Q

Given that the x coordinate of the point P is negative,

(c) using algebra and showing all stages of your working, find the coordinates of P

(5)

1. Find

$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) dx$$

giving your answer in simplest form.

(4)

blank

8. Solve, using algebra, the equation

$$x - 6x^{\frac{1}{2}} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

5.

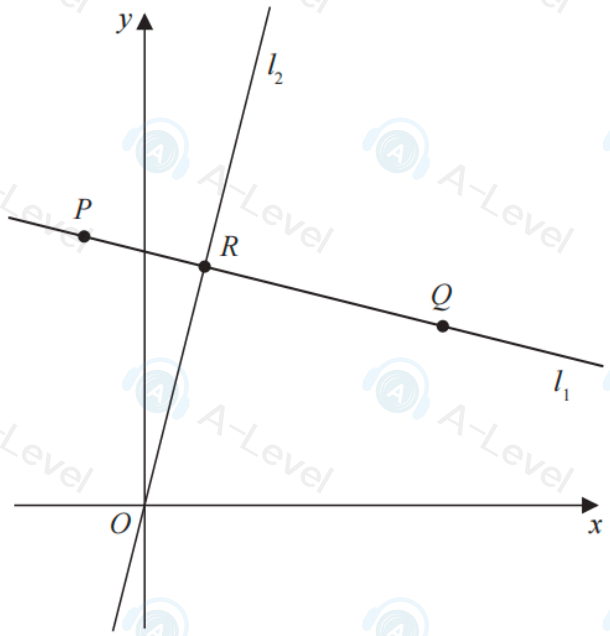

Figure 1

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The straight line l_1, shown in Figure 1, passes through the points $P(-2, 9)$ and $Q(10, 6)$.

(a) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The straight line l_2 passes through the origin O and is perpendicular to l_1

The lines l_1 and l_2 meet at the point R as shown in Figure 1.

(b) Find the coordinates of R

(4)

(c) Find the exact area of triangle OPQ.

(3)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{x+6}$$

State on your sketch

- the equation of the vertical asymptote
- the coordinates of the point of intersection of C with the y-axis

(3)

The straight line l has equation $y = mx - 4$, where m is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$9m^2 + 13m + 4 \geqslant 0$$

(ii) find the range of values of m.

(6)

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3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000

Exactly 10 years after monitoring began, the population was 65 000

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

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2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A vessel that contained some water started to leak from a hole in its base.

The volume of water in the vessel 25 minutes after the leak started was $6\,\text{m}^3$

The volume of water in the vessel 64 minutes after the leak started was $3.3\,\text{m}^3$

The volume of water, $V\,\text{m}^3$, in the vessel t minutes after the leak started, is modelled by the equation

$$V = a\sqrt{t} + b$$

where a and b are constants.

(a) Find the value of a and the value of b.

(4)

(b) Using the equation of the model,

(i) find the initial volume of water in the vessel,

(ii) find the time taken, after the leak started, for the vessel to empty. Give your answer to the nearest minute.

(3)

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4. Find

$$\int \frac{(3\sqrt{x} + 2)(x - 5)}{4\sqrt{x}}\,\mathrm{d}x$$

writing each term in simplest form.

(6)

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9.

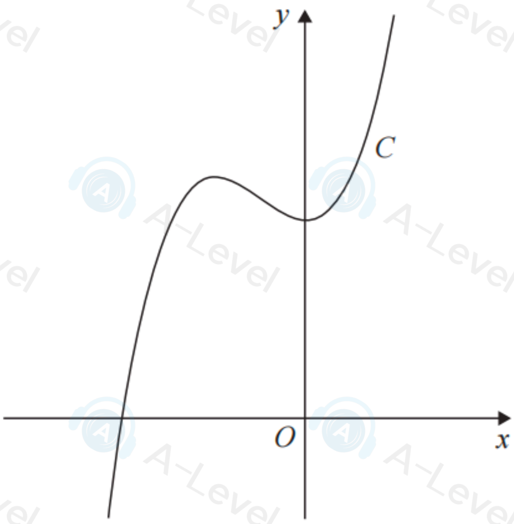

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Solve

$$5(x+3) > 4(2x-5)$$

(2)

(b) (i) Write

$$x^2 - 6x + 1$$

in the form $(x+a)^2 + b$ where a and b are constants.

(ii) Hence solve

$$x^2 - 6x + 1 \geqslant 0$$

(4)

(c) Hence find the values of x that satisfy both

$$5(x+3) > 4(2x-5) \quad \textbf{and} \quad x^2 - 6x + 1 \geqslant 0$$

(1)

5. A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

(3)

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

Leave blank

2. A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

where p and q are constants,

(a) find, to 3 significant figures where necessary, the value of p and the value of q. **(4)**

Exactly T years after the tree was planted, its height was 5 m.

(b) Find the value of T according to the model, giving your answer to one decimal place. **(2)**

DO NOT WRITE IN THIS AREA

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Using the laws of indices, solve

$$2^{4k-3} = \frac{8^{1-k}}{4\sqrt{2}}$$

(3)

(ii) Solve the equation

$$\frac{x\sqrt{3}+2}{\sqrt{3}-1} = x\sqrt{3} - 4$$

giving the answer in the form $a + b\sqrt{3}$, where a and b are rational numbers. **(4)**

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