4. The curve C_1 has equation

$$y = x^2 + kx - 9$$

and the curve C_2 has equation

$$y = -3x^2 - 5x + k$$

where k is a constant.

Given that C_1 and C_2 meet at a single point P

(a) show that

$$k^2 + 26k + 169 = 0$$

(3)

(b) Hence find the coordinates of P

(3)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{8 - \sqrt{15}}{2\sqrt{3} + \sqrt{5}}$ in the form $a\sqrt{3} + b\sqrt{5}$ where a and b are integers to be found.

(3)

(b) Hence, or otherwise, solve

$$\left(x + 5\sqrt{3}\right)\sqrt{5} = 40 - 2x\sqrt{3}$$

giving your answer in simplest form.

(3)

10.

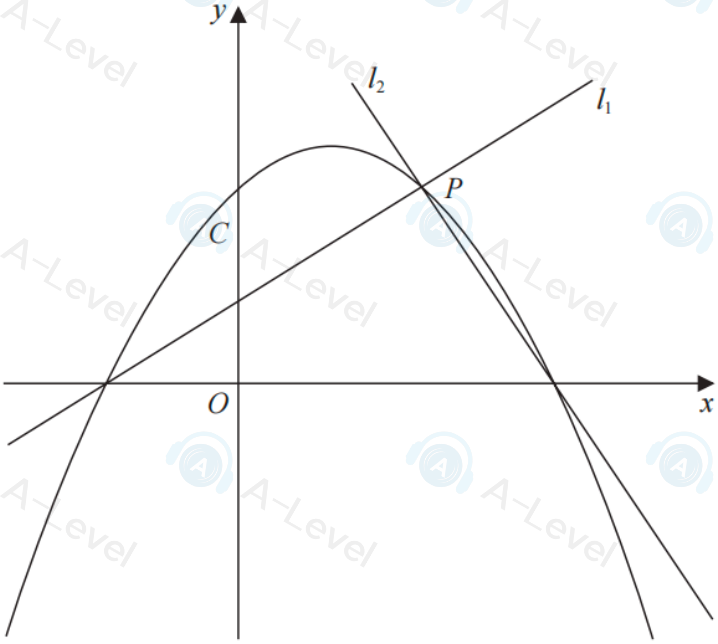

Figure 5

Figure 5 shows a sketch of the quadratic curve C with equation

$$y = -\frac{1}{4}(x+2)(x-b) \qquad \text{where } b \text{ is a positive constant}$$

The line l_1 also shown in Figure 5,

- has gradient $\frac{1}{2}$
- intersects C on the negative x-axis and at the point P

(a) (i) Write down an equation for l_1

(1)

(ii) Find, in terms of b, the coordinates of P

(3)

Given that the line l_2 is perpendicular to l_1 and intersects C on the positive x-axis,

(b) find, in terms of b, an equation for l_2

(2)

Given also that l_2 intersects C at the point P

(c) show that another equation for l_2 is

$$y = -2x + \frac{5b}{2} - 4$$

(2)

(d) Hence, or otherwise, find the value of b

(2)

11.

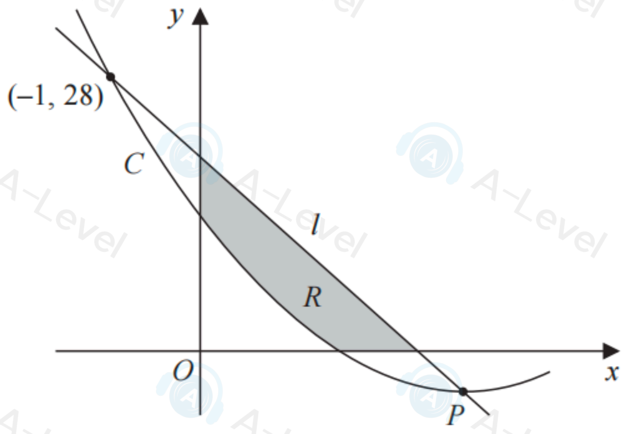

Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 3^x$, show that the equation

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

can be rewritten in the form

$$9p^2 + 26p - 3 = 0$$

(3)

(b) Hence solve

$$3 \times 9^x + 3^{x+2} = 1 + 3^{x-1}$$

(3)

5.

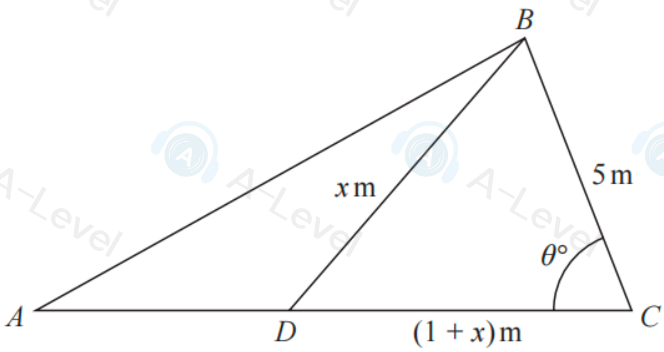

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle *ABD* joined to triangle *BCD*.

Given that

- $BD = x\,\text{m}$
- $CD = (1 + x)\,\text{m}$
- $BC = 5\,\text{m}$
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

(2)

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- ADC is a straight line

(b) find the area of triangle *ABC*, giving your answer, in m^2, to one decimal place.

(5)

3.

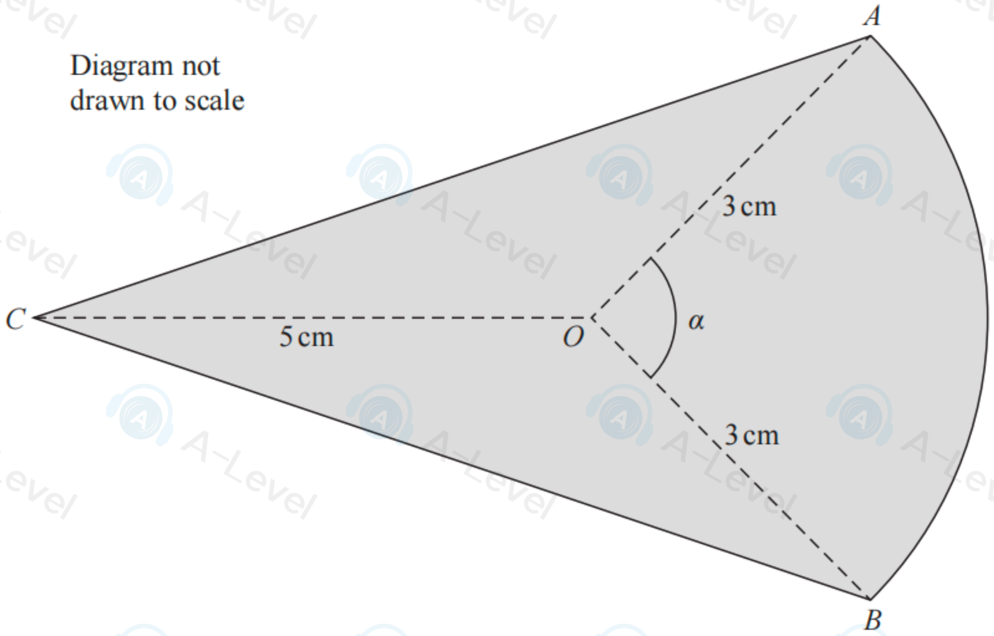

Figure 1

Figure 1 shows the design for a badge.

The design consists of two congruent triangles, AOC and BOC, joined to a sector AOB of a circle centre O.

- Angle $AOB = \alpha$
- $AO = OB = 3\,\text{cm}$
- $OC = 5\,\text{cm}$

Given that the area of sector AOB is $7.2\,\text{cm}^2$

(a) show that $\alpha = 1.6$ radians.

(2)

(b) Hence find

(i) the area of the badge, giving your answer in cm^2 to 2 significant figures,

(ii) the perimeter of the badge, giving your answer in cm to one decimal place.

(8)