

10. A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the x -axis at the point P and crosses the x -axis at the point Q .

- (a) State the coordinates of the point P .

(1)

- (b) Find $f'(x)$.

(4)

- (c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where k is a constant to be found.

(3)

The curve with equation $y = f(x + a)$, where a is a constant, passes through the origin O .

- (d) State the possible values of a .

(2)

3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (i)

$$f(x) = (x + \sqrt{2})^2 + (3x - 5\sqrt{8})^2$$

Express $f(x)$ in the form $ax^2 + bx\sqrt{2} + c$ where a , b and c are integers to be found.

(3)

- (ii) Solve the equation

$$\sqrt{3}(4y - 3\sqrt{3}) = 5y + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are simplified fractions to be found.

(4)

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int \left(\frac{2}{3}x^3 - \frac{1}{2x^3} + 5 \right) dx$$

simplifying your answer.

(4)

1. Find

$$\int \left(\frac{1}{2}x^3 + \frac{3}{\sqrt{x}} - 4 \right) dx$$

writing your answer in simplest form.

(4)

10. The curve C has equation $y = f(x)$ where $x > 0$

Given that

$$f'(x) = 6x - \frac{(2x-1)(3x+2)}{2\sqrt{x}}$$

- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P , giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $f(x)$, giving each term in simplest form.

(6)

2. (i) Given that $m = 2^n$, express each of the following in simplest form in terms of m .

(a) 2^{n+3}

(1)

(b) 16^{3n}

(2)

(ii) **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$x\sqrt{3} - 3 = x + \sqrt{3}$$

giving your answer in the form $p + q\sqrt{3}$ where p and q are integers.

(3)

4.

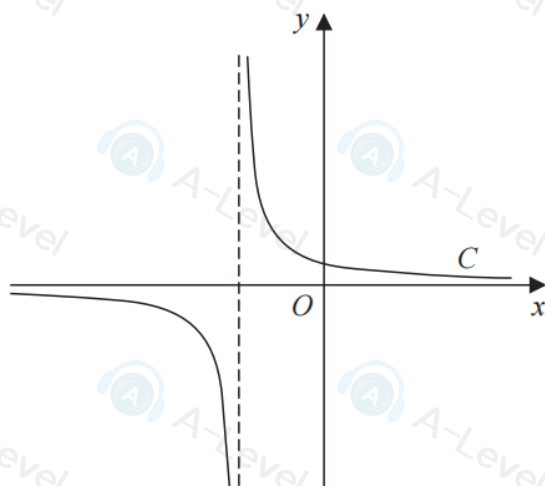


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \frac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y -axis.

(1)

(b) Factorise fully $x^3 + 4x^2 + 4x$

(2)

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

(3)

(d) Hence state the number of real solutions of the equation

$$(x+2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA