6.

Diagram not drawn to scale

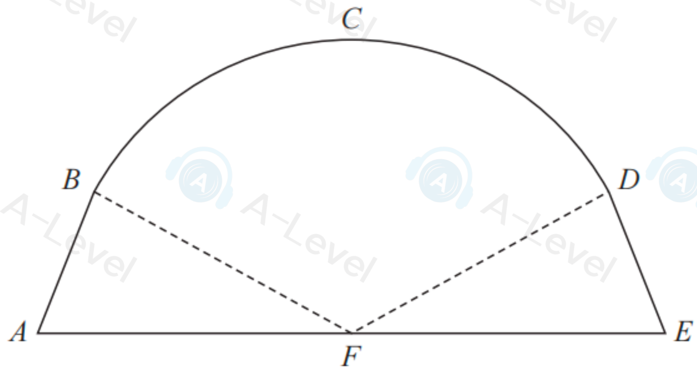

Figure 1

Figure 1 shows a sketch of the entrance to a tunnel.

The shape of the entrance consists of a sector $BCDF$, of a circle centre F, joined to two congruent (identical) triangles ABF and EDF.

Given that

AFE is a straight line

$AF = FE = 6.4\,\text{m}$

$FB = FD = 6.2\,\text{m}$

angle $BFD = 2.275$ radians

(a) Show that angle $AFB = 0.433$ radians to 3 decimal places.

(1)

(b) Find the perimeter of the entrance to the tunnel, $ABCDEFA$, in metres, to one decimal place.

(4)

(c) Find the cross-sectional area of the entrance to the tunnel, $ABCDEFA$, in m^2, to one decimal place.

(4)

DO NOT WRITE IN THIS AREA

10. The curve C has equation $y = \text{f}(x)$ where $x > 0$

Given that

- $\text{f}'(x) = 6x - \dfrac{(2x-1)(3x+2)}{2\sqrt{x}}$
- the point $P(4, 12)$ lies on C

(a) find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found,

(4)

(b) find $\text{f}(x)$, giving each term in simplest form.

(6)

DO NOT WRITE IN THIS AREA

11.

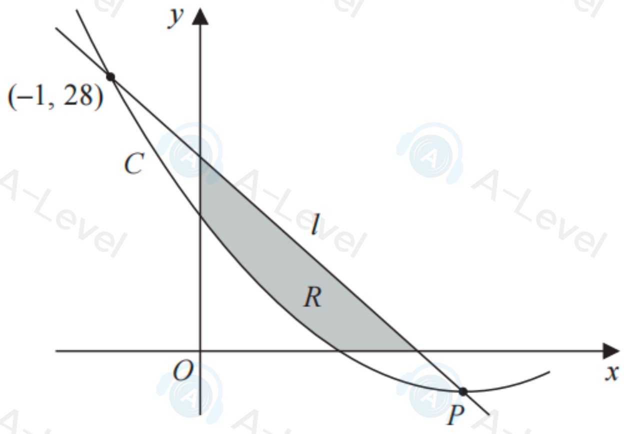

Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE

1. Find, in simplest form,

$$\int \left(\frac{8x^3}{3} - \frac{1}{2\sqrt{x}} - 5 \right) \mathrm{d}x$$

(4)

blank

DO NOT

blank

6. The line with equation $y = 4x + c$, where c is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(a) Find the value of c. **(4)**

(b) Hence find the coordinates of the point of intersection. **(3)**

DO NOT WRITE IN

1. A curve C has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form. **(3)**

(b) Hence find the exact value of the gradient of the tangent to C at the point where $x = 2$ giving your answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*) **(2)**

DO NOT WRITE IN THIS AREA

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The equation

$$4(p - 2x) = \frac{12 + 15p}{x + p} \qquad x \neq -p$$

where p is a constant, has two distinct real roots.

(a) Show that

$$3p^2 - 10p - 8 > 0$$

(3)

(b) Hence, using algebra, find the range of possible values of p **(3)**

DO NOT WRITE IN THIS AREA

1. Find

$$\int (2x - 5)(3x + 2)(2x + 5)\,dx$$

writing your answer in simplest form. **(5)**

DO NOT WRIT

5.

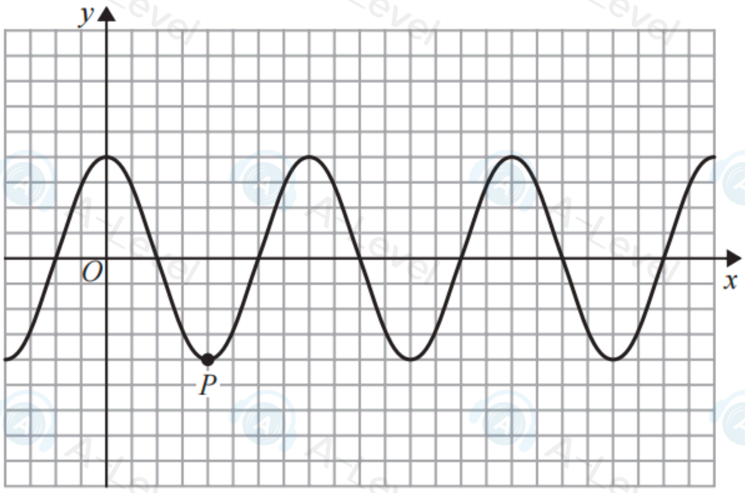

Figure 2

Figure 2 shows a plot of part of the curve with equation $y = \cos 2x$ with x being measured in radians.

The point P, shown on Figure 2, is a minimum point on the curve.

(a) State the coordinates of P.

(2)

A copy of Figure 2, called Diagram 1, is shown at the top of the next page.

(b) Sketch, on Diagram 1, the curve with equation $y = \sin x$

(2)

(c) Hence, or otherwise, deduce the number of solutions of the equation

(i) $\cos 2x = \sin x$ that lie in the region $0 \leqslant x \leqslant 20\pi$

(ii) $\cos 2x = \sin x$ that lie in the region $0 \leqslant x \leqslant 21\pi$

(2)

Question 5 continued

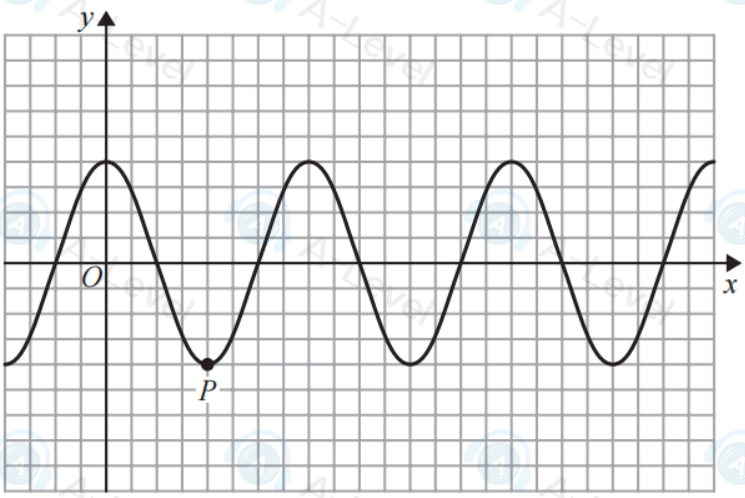

Diagram 1

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Solve

$$5(x + 3) > 4(2x - 5)$$

(2)

(b) (i) Write

$$x^2 - 6x + 1$$

in the form $(x + a)^2 + b$ where a and b are constants.

(ii) Hence solve

$$x^2 - 6x + 1 \geqslant 0$$

(4)

(c) Hence find the values of x that satisfy both

$$5(x + 3) > 4(2x - 5) \quad \textbf{and} \quad x^2 - 6x + 1 \geqslant 0$$

(1)

9.

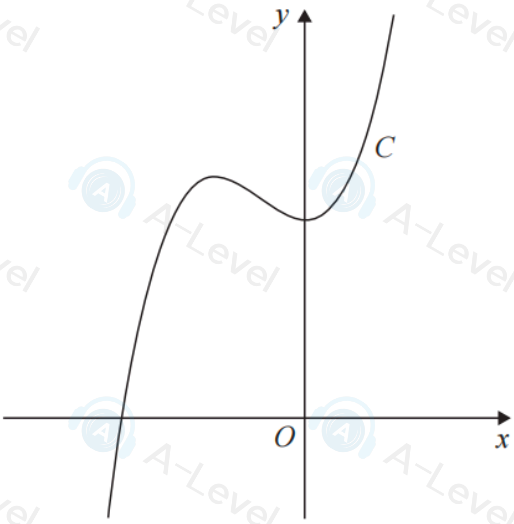

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x+5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x-3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

3.

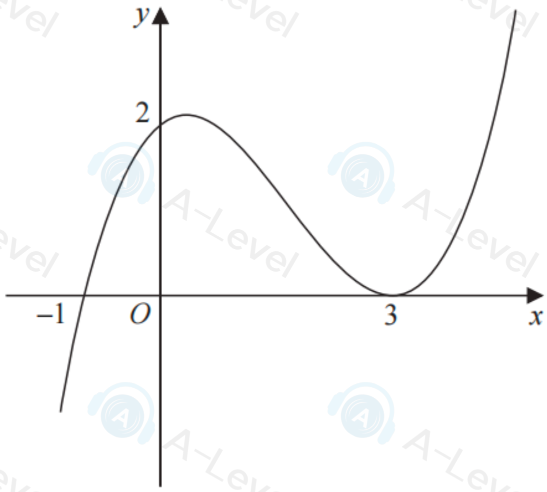

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-1, 0)$ and $(0, 2)$ and touches the x-axis at the point $(3, 0)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x + 3)$

(3)

(b) $y = f(-3x)$

(3)

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

6.

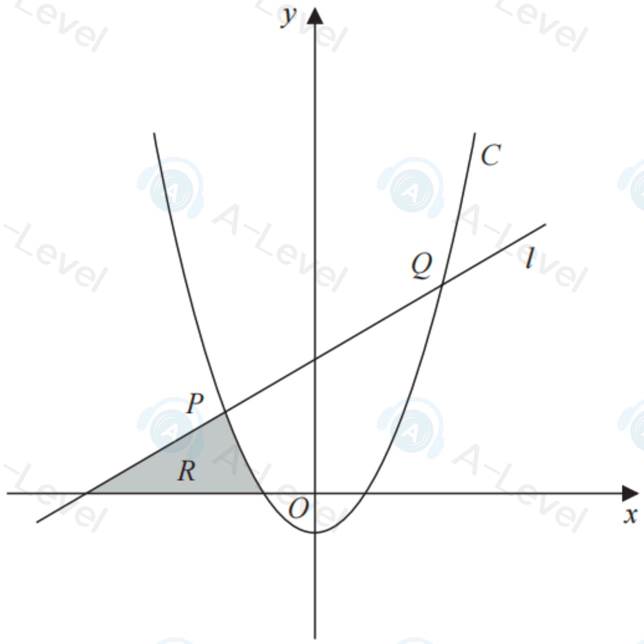

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows

- the line l with equation $y - 5x = 75$
- the curve C with equation $y = 2x^2 + x - 21$

The line l intersects the curve C at the points P and Q, as shown in Figure 3.

(a) Find, using algebra, the coordinates of P and the coordinates of Q.

(4)

The region R, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Use inequalities to define the region R.

(3)