Question	Scheme	Marks
9(a)	$V = hl^2 \Rightarrow 250000 = hl^2$ or $l = \dfrac{500}{\sqrt{h}}$ oe (may use e.g. $hl = \sqrt{250\,000h}$)	B1
	$S = l^2 + 4hl = \dfrac{"250000"}{h} + 4h \times \dfrac{"500"}{\sqrt{h}}$	M1
	$S = \dfrac{250000}{h} + 2000\sqrt{h}$ *	A1*
		(3)
(b)	$\dfrac{dS}{dh} = -\dfrac{250000}{h^2} + 2000 \times \dfrac{1}{2} h^{-\frac{1}{2}}$ oe	M1A1
	$\dfrac{dS}{dh} = 0 \Rightarrow -\dfrac{250000}{h^2} + 2000 \times \dfrac{1}{2} h^{-\frac{1}{2}} = 0 \Rightarrow h^k = \ldots$	dM1
	$\Rightarrow h^{\frac{3}{2}} = 250 \Rightarrow h = \ldots$	ddM1
	$h = 250^{\frac{2}{3}}$	A1
		(5)
(c)	$\dfrac{d^2S}{dh^2} = \dfrac{500000}{h^3} - 500h^{-\frac{3}{2}}$	M1
	$\left.\dfrac{d^2S}{dh^2}\right\|_{h=39.7} = 6 > 0$ hence gives the minimum value.	A1
		(2)
		(10 marks)

Question Number	Scheme	Notes	Marks
9(a)	$mx = x - x^2 \Rightarrow m = 1 - x \Rightarrow x = ...$ Or $y = \frac{y}{m} - \frac{y^2}{m^2} \Rightarrow m^2 = m - y \Rightarrow y = ...$	Attempts to eliminate either *x* or *y* and factors out or cancels *x*/*y* to get a linear equation and solve.	M1
	$x = 1 - m$ **and** $y = m(1-m)$	Both correct	A1
			(2)
(b)	$\int x - x^2 (-mx)\,dx = \frac{x^2}{2} - \frac{x^3}{3}\left(-m\frac{x^2}{2}\right)$	$x^n \to x^{n+1}$ in at least one term	M1
	Area of $R_1 = \int_0^{"1-m"} \{x - x^2(-mx)\}\,dx$ $= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3}\left(-m\frac{(1-m)^2}{2}\right) - 0$	Uses the limits "1 – *m*" and 0 in their integrated expression and subtracts (condone the omission of the "– 0")	M1
		Correct expression in *m* with/without the area under line subtracted.	A1
	Area of $R_1 = \int_0^{"1-m"} \{x - x^2 - mx\}\,dx = \frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3}(-0)$ Correct strategy for the area (may be scored for finding separate areas and subtracting)		dM1
	$= \frac{(1-m)^3}{6}$ *	Correct expression	A1*
			(5)
(c)	Area of $(R_1 + R_2) = \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = ...$ $\left(= \frac{1}{6}\right)$	Correct method for finding the area of $R_1 + R_2$ Alternatively, a correct method for finding the area of R_2	M1
	Alt: Area of $R_2 = \int_{1-m}^1 (x - x^2)\,dx + \frac{1}{2}("1-m") \times m(1-m)$ $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1-m}^1 + \frac{1}{2}m(1-m)^2 = ...\left(= \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2}m(1-m)^2\right)$		
	$R_1 = R_2 \Rightarrow \frac{(1-m)^3}{6} = \frac{1}{12} \Rightarrow m = ...$	Sets up a correct equation using the answer to part (b) and solves for *m*	dM1
	Alt: $\frac{(1-m)^3}{6} = \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2}m(1-m)^2 \Rightarrow m = ...$		
	$m = 1 - \frac{1}{\sqrt[3]{2}}$	Correct exact value in any form	A1
			(3)
			Total 10

Question Number	Scheme	Marks
	$y = \dfrac{9x^2\left(5-\sqrt{x}\right)}{5}$	
9(a)	$y = 9x^2 - \dfrac{9}{5}x^{\frac{5}{2}} \Rightarrow \dfrac{dy}{dx} = 18x - \dfrac{9}{2}x^{\frac{3}{2}}$	M1, A1
	$18x - \dfrac{9}{2}x^{\frac{3}{2}} = 0 \Rightarrow x^{\frac{1}{2}} = (4)$	dM1
	$(16, 460.8)$	A1, A1
		(5)
(b)	$5 - \sqrt{x} = 0 \Rightarrow x = 25$	M1, A1
		(2)
(c)	$\int 9x^2 - \dfrac{9}{5}x^{\frac{5}{2}}\,dx = 3x^3 - \dfrac{18}{35}x^{\frac{7}{2}}$	M1A1
	Shaded area = $\underline{(25-16)\times 460.8} - \left[3x^3 - \dfrac{18}{35}x^{\frac{7}{2}}\right]_{16}^{25}$	M1, dM1
	$=1312.7$	A1
		(5)
		(12 marks)

Question Number	Scheme	Marks
2(a)	$(S=)\ 6x^2+6xh+2xh$	B1
	eg $V=3x^2h=972 \Rightarrow h=\dfrac{972}{3x^2}$ or eg $hx=\dfrac{324}{x}$ $\Rightarrow (S=)\ 6x^2+8x\left(\dfrac{972}{3x^2}\right)$ or $\Rightarrow (S=)\ 6x^2+8\left(\dfrac{324}{x}\right)$	M1
	$S=6x^2+\dfrac{2592}{x}$ *	A1*
		(3)
(b)	$\left(\dfrac{dS}{dx}=\right) 12x-\dfrac{2592}{x^2}$	B1
		(1)
(c)	$12x-\dfrac{2592}{x^2}=0 \Rightarrow 12x^3=2592$ $\Rightarrow x=\sqrt[3]{\dfrac{2592}{12}}$	M1
	$x=6$	A1
		(2)
(d)	$\left(\dfrac{d^2S}{dx^2}=\right) 12+\dfrac{5184}{x^3}$	B1ft
	$\dfrac{d^2S}{dx^2}>0$ when $x=6$ so minimum	B1
		(2)
(e)	$S=6(6)^2+\dfrac{2592}{6}=648\ (\text{cm}^2)$	B1
		(1)
		Total 9

Question Number	Scheme	Marks
8 (i)	$S_n=a+ar+ar^2+\dots\dots ar^{n-1}$	B1
	$S_n=a+ar+ar^2+\dots\dots ar^{n-1}$ **and** $rS=ar+ar^2+ar^3+\dots\dots ar^n$	
	Finds S_n and rS_n and subtracts. E.g. $S-rS=\dots$	M1
	Completes proof $\Rightarrow S_n(1-r)=a(1-r^n) \Rightarrow S_n=\dfrac{a(1-r^n)}{(1-r)}$ *	A1*
		(3)
(ii) (a)	Attempts $U=150\times 0.92^n$ with $n=5$ or 6	M1
	$\Rightarrow (U_6)=150\times(0.92)^6=90.95$ (≈ 91 litres) *	A1*
		(2)
(b)	Attempts $S_n=\dfrac{a(1-r^n)}{(1-r)}$ with $n=39/40/41$ $a=150/138$ and $r=0.92$	M1
	$S=\dfrac{138(1-0.92^{40})}{(1-0.92)}$, $S=150\times\dfrac{0.92(1-0.92^{40})}{(1-0.92)}$ OR $S=\dfrac{150(1-0.92^{41})}{(1-0.92)}-150$	A1
	1664 litres	A1
		(3)
		(8 marks)

Question Number	Scheme	Marks
7. (i)	$10+8+6.4+...$	
(a)	Attempts $S_\infty = \frac{a}{1-r}$ with $a=10,\ r=\frac{8}{10}$	M1
	$S_\infty = 50$	A1
		(2)
(b)	Sets $10\times0.8^{k-1} < 0.0005 \Rightarrow 0.8^{k-1} < 0.00005$	M1
	e.g. $\Rightarrow k-1 > \frac{\log(0.00005)}{\log(0.8)}$ or e.g. $k-1 > \log_{0.8} 0.00005$ (see notes)	M1
	$\Rightarrow k-1 > 44.38 \Rightarrow (k=)\ 46$	A1
		(3)
(ii)	Sets $850+(n-1)\times -7 = 0 \Rightarrow n = 122.4$	M1
	States or implies that $n=122$	A1
	$S=\frac{122}{2}\{2\times850+121\times-7\}=52033$	dM1, A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
3.(a)	$300000 = 3\times2^{-k} \Rightarrow 2^{-k} = 100000 \Rightarrow -k = \frac{\log 100000}{\log 2}$	M1
	$(k) = -16.61$	A1
		(2)
(b) (i)	Strip width = 1.5	B1
	$\frac{1.5}{2}\{4.243+0.023+2(1.5+0.530+0.188+0.066)\} = 6.63$	M1, A1
		(3)
(ii)	$\int_{-0.5}^{7} 2^{-x}\,dx + \int_{-7}^{0.5} 2^{x}\,dx = \frac{1}{3}\times"6.63"+\frac{1}{3}\times"6.63" = 4.42$	M1, A1
		(2)
		(7 marks)

Question Number	Scheme	Marks
4	$\int\left(2x^2+7\right)dx = \frac{2}{3}x^3+7x$ or $\int\left(10-2x^2\right)dx = 10x-\frac{2}{3}x^3$	M1 A1
	Achieves/uses a limit of $\sqrt{5}$	B1
	Area $= 17\sqrt{5} - \int_0^{\sqrt{5}}\left(2x^2+7\right)dx$ | Area $= \int_0^{\sqrt{5}}\left(10-2x^2\right)dx$	M1
	$= 17\sqrt{5}-\frac{2}{3}\times5\sqrt{5}-7\sqrt{5}$ | $=10\sqrt{5}-\frac{2}{3}\times5\sqrt{5}$	M1
	$=\frac{20}{3}\sqrt{5}$	A1
		(6)
		(6 marks)

Question Number	Scheme	Marks
8 .(i)	States $(S=)\ a+\ (a+d)\ +\ldots\ldots\{a+(n-2)d\}+\{a+(n-1)d\}$	B1
	$(S=)\{a+(n-1)d\}+\{a+(n-2)d\}+\ldots\ldots(a+d)\ +a$ and adds	M1
	$2S=n(2a+(n-1)d)\Rightarrow S=\frac{n}{2}\{2a+(n-1)d\}$ *	A1*
		(3)
(ii)	(a) $u_5=22$	B1
	(b) $\sum_{n=1}^{59}u_n=(5+10+15+\ldots)+(-3+3-3+\ldots)$	
	$=\frac{59}{2}\{10+58\times5\}+(-3)=8850-3=8847$	M1 B1 A1
		(4)
		(7 marks)

Question Number	Scheme	Marks
6(a)	Sets $\mathrm{f}\left(-\frac{3}{2}\right)=0\Rightarrow(9p+4q=102)$	M1
	Sets $\mathrm{f}(-2)=-5\Rightarrow(4p+q=43)$	M1
	Solves to get values for p and q	dM1
	(i) $p=10$ * (ii) $q=3$ following two correct equations	A1*, A1
		(5)
(b)	$\mathrm{f}'(x)=12x^2+20x+8$	B1
	Solves $\mathrm{f}'(x)=0\Rightarrow4(3x+2)(x+1)=0\Rightarrow x=-\frac{2}{3},-1$	M1, A1
	$-1<x<-\frac{2}{3}$	A1
		(4)
		Total 9

Question	Answer	Marks	Guidance
6(a)	Obtain at least either $(\frac{1}{2}\sin\theta+\frac{1}{2}\sqrt{3}\cos\theta)$ or $(\frac{1}{2}\cos\theta+\frac{1}{2}\sqrt{3}\sin\theta)$	B1	Allow if implied by decimal values.
	Expand and simplify with correct use of $\sin^2\theta+\cos^2\theta=1$	M1	
	Use $\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta$	M1	
	Confirm given result $\sqrt{3}+2\sin 2\theta$	A1	AG necessary detail required.
		4	
6(b)	Identify value of θ is $\frac{3}{8}\pi$	*B1	OE
	Obtain $\sqrt{3}+2\sin\frac{3}{4}\pi$ and conclude $\sqrt{3}+\sqrt{2}$	DB1	or exact equivalent.
		2	
6(c)	Identify integrand as $\sqrt{3}+2\sin 4x$	B1	
	Integrate to obtain form $k_1x+k_2\cos 4x$	M1	where $k_1k_2\neq 0$.
	Obtain correct $\sqrt{3}x-\frac{1}{2}\cos 4x$	A1	
	Obtain $\frac{1}{8}\pi\sqrt{3}+\frac{1}{2}$	A1	or exact equivalent.
		4	

Question Number	Scheme	Marks
4. (a)	$\log_3(a+1)-\log_3 a=4\Rightarrow\log_3\left(\frac{a+1}{a}\right)=4$	M1
	$\Rightarrow\left(\frac{a+1}{a}\right)=3^4$	A1
	$\Rightarrow a+1=81a\Rightarrow a=\frac{1}{80}$	dM1, A1
		(4)
(b)	Area $\approx\frac{1}{2}\left\{\log_3\left(\frac{2}{1}\right)+\log_3\left(\frac{6}{5}\right)+2\left(\log_3\left(\frac{3}{2}\right)+\log_3\left(\frac{4}{3}\right)+\log_3\left(\frac{5}{4}\right)\right)\right\}$	M1, A1
	$=\frac{1}{2}\log_3\left(\frac{2}{1}\times\frac{6}{5}\times\frac{3^2}{2^2}\times\frac{4^2}{3^2}\times\frac{5^2}{4^2}\right)=\frac{1}{2}\log_3 15=\log_3\sqrt{15}$	dM1, A1
		(4)
(c)	States ‘increase the number of strips’	B1
		(1)
		(9 marks)

Question Number	Scheme	Marks
7(a)	$r=\sqrt{\frac{12.8}{20}}=0.8*$ or e.g. $20\times0.8\times0.8=12.8$ so $r=0.8*$	B1*
		(1)
(b)	$a=20\div0.8^2$	M1
	$=31.25$	A1
		(2)
(c)	$\frac{31.25(1-0.8^n)}{1-0.8}>156$	M1
	Eg $1-0.8^n>0.9984$ $\Rightarrow 0.8^n<0.0016$	dM1
	$0.8^n<0.0016\Rightarrow n>\frac{\log(0.0016)}{\log(0.8)}$ or $0.8^n<0.0016\Rightarrow n>\log_{0.8}0.0016$	M1
	$n=29$	A1
		(4)
		Total 7