Question Number	Scheme	Marks
9(a)	$\left(2\tan\theta = 3\cos\theta \Rightarrow\right)\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta = 3\left(1-\sin^2\theta\right)$	M1
	$2\sin\theta = 3\left(1-\sin^2\theta\right) \Rightarrow 3\sin^2\theta + 2\sin\theta - 3 = 0$*	A1*
		(3)
(b)	$\left(\sin\left(2x+\dfrac{\pi}{3}\right)=\right)\dfrac{-1\pm\sqrt{10}}{3}$ (May only see positive root) NB decimal roots are: $-1.387\ldots$, $0.7207\ldots$	M1
	$2x+\dfrac{\pi}{3} = \sin^{-1}(0.7207\ldots) \Rightarrow x = \ldots$	M1
	$-0.121, -2.50, 0.645, 3.02$	A1A1
		(4)
		Total 7

Question Number	Scheme	Marks
1 (a)	$y = 2x^2(x-5) = 2x^3 - 10x^2$	B1
	$\dfrac{dy}{dx} = 6x^2 - 20x$	M1
	Sets $\dfrac{dy}{dx} = 0 \Rightarrow 6x^2 - 20x = 0 \Rightarrow x = 0, \dfrac{10}{3}$ oe	dM1 A1
		(4)
(b)	$x \leqslant 0, \quad x \geqslant \dfrac{10}{3}$	M1 A1
		(2) **(6 marks)**
Alt (a)	$\left(\dfrac{d(uv)}{dx} = uv' + vu'\right) \quad u = 2x^2, u' = 4x, v = x-5, v' = 1$	B1
	$\left(\dfrac{dy}{dx} =\right) Ax(x-5) + Bx^2$	M1
	Sets $\dfrac{dy}{dx} = 0 \Rightarrow 4x(x-5) + 2x^2 = 0 \Rightarrow x = 0, \dfrac{10}{3}$	dM1 A1

Question Number	Scheme	Marks
6 (i) (a)	Uses $\sin^2\theta+\cos^2\theta=1$ with $\cos\theta=\frac{1}{\sqrt{5}}\Rightarrow\sin^2\theta=\frac{4}{5}$	M1
	$\Rightarrow\sin\theta=-\frac{2}{\sqrt{5}}$	A1
(b)	Uses $\tan\theta=\frac{\sin\theta}{\cos\theta}$ with $\cos\theta=\frac{1}{\sqrt{5}}$ and their $\sin\theta=-\frac{2}{\sqrt{5}}$	M1
	$\Rightarrow\tan\theta=-2$	A1
		(4)
(ii) (a)	1 (m)	B1
		(1)
(b)	$30t-40=180\Rightarrow t=7.33$ Hence 7:20 **am**	M1, A1, A1
		(3)
(c)	$4+3\cos(30T-40)°=3.5\Rightarrow\cos(30T-40)°=-\frac{1}{6}$	M1
	$\Rightarrow 30T-40=99.6,\ 260.4,\ \underline{\underline{459.6}}$	A1
	$\Rightarrow T=\frac{\text{"}459.6\text{"}+40}{30}=16.65$	dM1, A1
		(4)
		(12 marks)

Question	Answer	Marks	Guidance
8(a)	Use $\sin 2\theta=2\sin\theta\cos\theta$ and $\sec\theta=\frac{1}{\cos\theta}$ to obtain $6\sin\theta$	**B1**	
	Expand second term to obtain $5\sqrt{3}\cos\theta+5\sin\theta$	**B1**	
	Simplify to obtain $11\sin\theta+5\sqrt{3}\cos\theta$	**B1**	
	State $R=14$	**B1 FT**	FT *their* $k_1\cos\theta+k_2\sin\theta$
	Use appropriate trigonometry to find α	**M1**	
	Obtain $\alpha=38.21$	**A1**	AWRT
		6	
8(b)	State or imply $14\sin(2\beta+38.21)=2$	**B1 FT**	FT *their* R and α
	Carry out correct process to find value of β between 0° and 90°	**M1**	
	Obtain 66.8	**A1**	AWRT
		3	

Question Number	Scheme	Marks
8 (i)	(a) $S_n = a+(a+d)+........+(a+(n-1)d)$ (1)	B1
	$S_n = (a+(n-1)d)+.................(a+d)+a$ (2)	M1
	(1) + (2) $2S_n = (2a+(n-1)d)+(2a+(n-1)d)........+(2a+(n-1)d)$	
	$2S_n = n(2a+(n-1)d) \Rightarrow S_n = \frac{n}{2}(2a+(n-1)d)$ *	A1*
		(3)
	(b) States or uses two of $a = 900$, $d = -8$ and $n = 51$ Or alternatively states or uses two of $a = 500$, $d = 8$ and $n = 51$	B1
	Complete method E.g $n = \frac{900-500}{8}+1$ and $S = \frac{51}{2}\{2\times 900+50\times -8\}$	M1
	35700	A1
		(3)
(ii)	(a) $\frac{11-k}{k-2} = \frac{k-2}{k+4}$	M1
	$44-k^2+7k = k^2-4k+4$	dM1
	$2k^2-11k-40=0$ *	A1*
		(3)
	(b) $2k^2-11k-40=0 \Rightarrow k = \left(-\frac{5}{2}\right), 8$	B1
	Uses either k value and attempts to find both a and r. E.g. with $k=8 \Rightarrow a = 8+4, r = \frac{8-2}{8+4}$	M1
	Uses $S_\infty = \frac{a}{1-r} = \frac{12}{1-\frac{1}{2}} = 24$	dM1, A1
		(4)
		Total 13

Question Number	Scheme	Marks
4(a)	$y = 4x^{\frac{1}{2}}+9x^{-\frac{1}{2}}+3$ $\Rightarrow \left(\frac{dy}{dx}=\right)2x^{-\frac{1}{2}}-\frac{9}{2}x^{-\frac{3}{2}}$	M1A1
		(2)
(b)	$\frac{dy}{dx}=0 \Rightarrow 2x^{-\frac{1}{2}}-\frac{9}{2}x^{-\frac{3}{2}}=0 \Rightarrow 4x-9=0 \Rightarrow x=...$	M1
	$x=\frac{9}{4}$ oe e.g. 2.25	A1
		(2)

4(c)(i)	$\left(\frac{d^2y}{dx^2}=\right)-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}}+6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{d^2y}{dx^2}\right)_{x=\frac{9}{4}}=\right)-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}\left(=\frac{16}{27}(0.5925...)\right)$ $\left(\frac{d^2y}{dx^2}\right)>0$ so (local) minimum	B1
		(2)
(d)	$0<x<\frac{9}{4}$	B1ft
		(1)
		Total 7

Question Number	Scheme	Marks
2(a)	$(S=)\ 6x^2+6xh+2xh$	B1
	eg $V=3x^2h=972\Rightarrow h=\frac{972}{3x^2}$ or eg $hx=\frac{324}{x}$ $\Rightarrow(S=)\ 6x^2+8x\left(\frac{972}{3x^2}\right)$ or $\Rightarrow(S=)\ 6x^2+8\left(\frac{324}{x}\right)$	M1
	$S=6x^2+\frac{2592}{x}$ *	A1*
		(3)
(b)	$\left(\frac{dS}{dx}=\right)12x-\frac{2592}{x^2}$	B1
		(1)
(c)	$12x-\frac{2592}{x^2}=0\Rightarrow 12x^3=2592$ $\Rightarrow x=\sqrt[3]{\frac{2592}{12}}$	M1
	$x=6$	A1
		(2)
(d)	$\left(\frac{d^2S}{dx^2}=\right)12+\frac{5184}{x^3}$	B1ft
	$\frac{d^2S}{dx^2}>0$ when $x=6$ so minimum	B1
		(2)
(e)	$S=6(6)^2+\frac{2592}{6}=648\ (\text{cm}^2)$	B1
		(1)
		Total 9

Question Number	Scheme	Marks
7 (i)	$(2\theta=)\arctan\left(\frac{5}{7}\right)$	M1A1
	$(\theta=)$ awrt $17.8°, 107.8°$	dM1A1
		(4)
(ii)	$24\tan x=5\cos x\Rightarrow 24\sin x=5\cos^2 x$ oe	M1
	$\Rightarrow 24\sin x=5(1-\sin^2 x)$	dM1
	$\Rightarrow 5\sin^2 x+24\sin x-5=0$	A1
	$\Rightarrow \sin x=\frac{1}{5}$	ddM1
	$\Rightarrow (x=)$ awrt $0.201, 2.940$	A1
		(5)
		(9 marks)

Question Number	Scheme	Marks
9 (i)	States or uses $\tan x=\frac{\sin x}{\cos x}$	B1
	$\sin x\tan x=5\Rightarrow \sin^2 x=5\cos x\Rightarrow 1-\cos^2 x=5\cos x$	M1A1
	$\cos^2 x+5\cos x-1=0\Rightarrow(\cos x=)\frac{-5\pm\sqrt{29}}{2}\Rightarrow x=$ awrt $78.9°, 281.1°$	M1dM1A1
		(6)
(ii)	(a) $A=5$	B1
		(1)
	(b) $2\theta-\frac{3\pi}{8}=\frac{3\pi}{2}\Rightarrow\theta=...$	M1
	$\theta=\frac{15\pi}{16}$	A1
	y coordinate $Q=-3$ (or $2-"A"$)	B1ft
		(3)
	(c) Sets $0="5"\sin\left(2\theta-\frac{3\pi}{8}\right)+2\Rightarrow\sin\left(2\theta-\frac{3\pi}{8}\right)=\pm\frac{2}{"5"}$	M1
	$\sin\left(2\theta-\frac{3\pi}{8}\right)=\pm\frac{2}{5}\Rightarrow\left(2\theta-\frac{3\pi}{8}\right)=\arcsin\left(\pm\frac{2}{5}\right)=...$	dM1
	One of $\theta=0.38,\ 2.4,\ 3.5,\ 5.5,\ 6.7,\ 8.6,\ 9.8....$	A1
	$\theta=$ awrt 5.51	A1
		(4)
		Total 14

Question	Answer	Marks	Guidance
5	Use product rule to differentiate $4e^{2x}y$	M1	
	Obtain correct $8e^{2x}y+4e^{2x}\frac{dy}{dx}$	A1	
	Obtain $\left[8e^{2x}y+4e^{2x}\frac{dy}{dx}\right]+2y\frac{dy}{dx}=0$	B1	
	Substitute $x=0$ and $y=-7$ to find value of $\frac{dy}{dx}$	M1	dependent at least one term involving $\frac{dy}{dx}$ from implicit differentiation.
	Obtain $-\frac{28}{5}$	A1	OE
		5	

Question Number	Scheme	Marks
10. (a)		
	Sets $30 = 2r^2 + 2r^2\theta + 2r^2\theta$	M1
	Obtains $4r\theta = \frac{30-2r^2}{r}$ oe or $\theta = \frac{30-2r^2}{4r^2}$ oe or $r\theta = \frac{30-2r^2}{4r}$ oe	A1
	Uses $P = 6r + 4r\theta$ with their θ or $...r\theta$ substituted	dM1
	$P = 6r + \frac{30-2r^2}{r} = 4r + \frac{30}{r}$ *	A1*
		(4)
(b)	Differentiates with $r^{-1} \to r^{-2}$	M1
	$\left(\frac{dP}{dr} =\right) 4 - \frac{30}{r^2}$	A1
	Solves $\frac{dP}{dr} = 0 \Rightarrow \left(r = \sqrt{\frac{15}{2}}\right)$ and substitutes to find P	dM1
	$\left(P = 4\sqrt{\frac{15}{2}} + \frac{30\sqrt{2}}{\sqrt{15}} = 2\sqrt{30} + 2\sqrt{30}\right) = 4\sqrt{30}$	A1
		(4)
(c)	e.g. finds $\left(\frac{d^2P}{dr^2} =\right) \frac{60}{r^3} = \frac{60}{"2.7"^3} = ...$	M1
	e.g. states that $\left(\frac{d^2P}{dr^2} =\right)$ awrt $3 > 0$ hence minimum	A1
		(2)
		(10 marks)

Question Number	Scheme		Marks
	OPEN TOPPED	**CLOSED CONTAINER**	
3.(a)	$120 = 3x^2 y \Rightarrow y = \dfrac{40}{x^2}$	$120 = 3x^2 y \Rightarrow y = \dfrac{40}{x^2}$	M1, A1
	$A = 3x^2 + 8xy$	$A = 6x^2 + 8xy$	
	$= 3x^2 + 8x \times \dfrac{40}{x^2} = 3x^2 + \dfrac{320}{x}$	$= 6x^2 + 8x \times \dfrac{40}{x^2} = 6x^2 + \dfrac{320}{x}$	dM1, A1
			(4)
(b)	$\dfrac{dA}{dx} = 6x - \dfrac{320}{x^2}$	$\dfrac{dA}{dx} = 12x - \dfrac{320}{x^2}$	M1, A1ft
	$\dfrac{dA}{dx} = 6x - \dfrac{320}{x^2} = 0$	$\dfrac{dA}{dx} = 12x - \dfrac{320}{x^2} = 0$	
	$\Rightarrow x^3 = \dfrac{160}{3} \Rightarrow x = \text{awrt } 3.76$	$\Rightarrow x^3 = \dfrac{80}{3} \Rightarrow x = \text{awrt } 2.99$	dM1, A1
			(4)
(c)	Attempts $\left.\dfrac{d^2A}{dx^2}\right\|_{x=3.76} = 6 + \dfrac{640}{x^3}$	Attempts $\left.\dfrac{d^2A}{dx^2}\right\|_{x=2.99} = 12 + \dfrac{640}{x^3}$	M1
	$= 6 + \dfrac{640}{3.76^3} = ..$	$= 12 + \dfrac{640}{2.99^3} = ..$	
	$\left.\dfrac{d^2A}{dx^2}\right\|_{x=3.76} = \text{awrt } 18 > 0$ MINIMUM*	$\left.\dfrac{d^2A}{dx^2}\right\|_{x=2.99} = \text{awrt } 36 > 0$ MINIMUM*	A1*
			(2)
			(10 marks)

Question Number	Scheme	Marks
4 (a)	$f(x) = 4x^3 + 13x^2 - 10x + 8$	
	Long division: $\begin{array}{r} 4x^2 + 21x + 32 \\ x-2\,\overline{)\,4x^3 + 13x^2 - 10x + 8} \\ \underline{4x^3 - 8x^2} \\ 21x^2 - 10x \\ \underline{21x^2 - 42x} \\ 32x + 8 \\ \underline{32x - 64} \\ 72 \end{array}$ Synthetic Division: $\begin{array}{c\|cccc} 2 & 4 & 13 & -10 & 8 \\ & & 8 & 42 & 64 \\ \hline & 4 & 21 & 32 & 72 \end{array}$	
(i)	$Q(x) = 4x^2 + 21x + 32$	M1, A1
(ii)	$R = 72$	M1, A1
		(4)
(b)(i)	Attempts $f(-4) = 4\times -64 + 13\times 16 - 10\times -4 + 8$	M1
	$= -256 + 208 + 40 + 8 = 0$ Hence $(x+4)$ is a factor *	A1*
(ii)	$f(x) = 4x^3 + 13x^2 - 10x + 8 = (x+4)(4x^2 - 3x + 2)$	M1
	For their $4x^2 - 3x + 2$ attempts "$b^2 - 4ac$" $= 9 - 32$,	M1
	$b^2 - 4ac' < 0$ so $4x^2 - 3x + 2$ has no real roots and $f(x) = 0$ has one at $x = -4$	A1*
		(5)
(c)	$(f'(x)) = 12x^2 + 26x - 10 = 2(3x-1)(2x+5)$	M1
	$-\frac{5}{2} < x < \frac{1}{3}$	dM1, A1
		(3)
		(12 marks)