

| Question Number | Scheme | Marks |
|-----------------|--|-----------------------------------|
| 4. (a) | $\log_3(a+1) - \log_3 a = 4 \Rightarrow \log_3\left(\frac{a+1}{a}\right) = 4$ $\Rightarrow \left(\frac{a+1}{a}\right) = 3^4$ $\Rightarrow a+1 = 81a \Rightarrow a = \frac{1}{80}$ | M1 A1 dM1, A1 (4) |
| (b) | $\text{Area} \approx \frac{1}{2} \left\{ \log_3\left(\frac{2}{1}\right) + \log_3\left(\frac{6}{5}\right) + 2 \left(\log_3\left(\frac{3}{2}\right) + \log_3\left(\frac{4}{3}\right) + \log_3\left(\frac{5}{4}\right) \right) \right\}$ $= \frac{1}{2} \log_3\left(\frac{2}{1} \times \frac{6}{5} \times \frac{3^2}{2^2} \times \frac{4^2}{3^2} \times \frac{5^2}{4^2}\right) = \frac{1}{2} \log_3 15 = \log_3 \sqrt{15}$ | M1, A1 dM1, A1 (4) |
| (c) | States 'increase the number of strips' | B1 (1) |
| | | (9 marks) |

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|-----------------|---|----------------|
| 10 (a) | $\left(\frac{dy}{dx}\right) x - 2187x^{-\frac{5}{2}}$ | M1, A1 |
| | Sets $x - 2187x^{-\frac{5}{2}} = 0 \Rightarrow x^{\frac{7}{2}} = 2187$ (or e.g. $x = (\sqrt[7]{2187})^2 \Rightarrow x = 9$ * | dM1A1* |
| | | (4) |
| (b) | e.g. $\int \left\{ \frac{1}{2}x^2 + 1458x^{-\frac{3}{2}} - 74 \right\} dx = \frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x$ or $\int \left\{ \frac{1}{2}x^2 + 1458x^{-\frac{3}{2}} - "94.5" \right\} dx = \frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 94.5x$ | M1A1ft |
| | y value at P is 20.5 | B1 |
| | e.g. Area R = $\left[\frac{1}{6}x^3 - 2916x^{-\frac{1}{2}} - 74x \right]_4^9 - (9-4) \times "20.5"$ $= \left(\frac{1}{6} \times 9^3 - 2916 \times 9^{-\frac{1}{2}} - 74 \times 9 \right) - \left(\frac{1}{6} \times 4^3 - 2916 \times 4^{-\frac{1}{2}} - 74 \times 4 \right) - (9-4) \times "20.5"$ | dM1 |
| | $\left(-1516\frac{1}{2} + 1743\frac{1}{3} - 102\frac{1}{2} \right) = 124\frac{1}{3}$ | A1 |
| | | (5) |
| | | Total 9 |

| Question Number | Scheme | Marks |
|-----------------|--|----------------------|
| 6(a) | Sets $f\left(-\frac{3}{2}\right) = 0 \Rightarrow (9p + 4q = 102)$ | M1 |
| | Sets $f(-2) = -5 \Rightarrow (4p + q = 43)$ Solves to get values for p and q (i) $p = 10$ * (ii) $q = 3$ following two correct equations | M1 dM1 A1*, A1 |
| | | (5) |
| (b) | $f'(x) = 12x^2 + 20x + 8$ | B1 |
| | Solves $f'(x) = 0 \Rightarrow 4(3x + 2)(x + 1) = 0 \Rightarrow x = -\frac{2}{3}, -1$ $-1 < x < -\frac{2}{3}$ | M1, A1 A1 |
| | | (4) |
| | | Total 9 |

| Question Number | Scheme | Notes | Marks |
|-----------------|--|---|------------|
| 4 | $2 \log_3(1-x) = \log_3(1-x)^2$ or $3 = \log_3 3^3$ | Correct power law used or implied | B1 |
| | $\log_3(32-12x) - \log_3(1-x)^2 = \log_3 \frac{32-12x}{(1-x)^2}$ Combines 2 log terms correctly | | M1 |
| | $\frac{32-12x}{(1-x)^2} = 27$ | Obtains this equation in any form | A1 |
| | $\Rightarrow 27x^2 - 42x - 5 = 0 \Rightarrow x = \dots$ | Solves 3TQ | M1 |
| | $x = -\frac{1}{9}$ | This value only i.e. the $\frac{5}{3}$ must clearly be discarded if seen. | A1 |
| | | | (5) |
| | | Total 5 | |

| Question | Answer | Marks | Guidance |
|----------|---|-------|--|
| 5(a) | Substitute $x = \frac{1}{2}$ and $x = 3$, and equate each to zero to produce two equations | M1 | SC B1 for a correct equation, if M0 otherwise. |
| | Obtain $\frac{1}{16}a + \frac{1}{8}b = -\frac{3}{4}$ | A1 | OE $a + 2b + 12 = 0$ |
| | Obtain $81a + 27b = -27$ | A1 | OE $3a + b + 1 = 0$ |
| | Solve simultaneous equations to obtain $a = 2$ and $b = -7$ | A1 | |
| | | 4 | |
| 5(b) | Divide by $2x^2 - 7x + 3$ or successively by $2x - 1$ and $x - 3$ | M1 | OE method, such as inspection. |
| | Obtain quotient $x^2 + 5$ | A1 | Condone $2(x^2 + 5)$ from synthetic division. |
| | State fully factorised form $(2x - 1)(x - 3)(x^2 + 5)$ | A1 | |
| | | 3 | |
| 5(c) | Attempt solution of at least $\cot 2\theta = 3$ | M1 | |
| | Obtain $\tan 2\theta = \frac{1}{3}$ and hence $\theta = 0.161$ | A1 | Or greater accuracy 0.16087... |
| | | 2 | |

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|-----------------|---|-----------------|
| 10 (a) | $y = \frac{9x - x^2}{2\sqrt{x}} = \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} \Rightarrow \frac{dy}{dx} = \frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{\frac{1}{2}}$ | M1, A1 |
| | Sets $\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{\frac{1}{2}} = 0 \Rightarrow x = \frac{9}{4} \times \frac{4}{3} \Rightarrow x = 3$ | dM1, A1 |
| | | (4) |
| (b) | $\int \left\{ \frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{3}{2}} \right\} dx = 3x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}}$ | M1, A1 |
| | Upper limit is 9 Area $R = \left[3x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}} \right]_1^9 = \left(3 \times 27 - \frac{1}{5} \times 243 \right) - \left(3 \times 1 - \frac{1}{5} \times 1 \right)$ $= \frac{148}{5}$ | B1 dM1 A1 |
| | | (5) |
| | | Total 9 |

| Question | Answer | Marks | Guidance |
|----------|--|-------|---------------------------|
| 3 | Attempt use of quotient rule (or equivalent) to differentiate $\frac{8x}{2x+3}$ or $\frac{-12x^2+15}{(2x+3)}$ | *M1 | |
| | Obtain $\frac{dy}{dx} = \frac{24}{(2x+3)^2} - 6$ | A1 | OE Allow unsimplified. |
| | Equate first derivative to zero, and attempt solution of a quadratic equation to find two values of x | DM1 | $4x^2 + 12x + 5 = 0$ |
| | Obtain at least one of the stationary points $(-\frac{5}{2}, 30)$ and $(-\frac{1}{2}, 6)$, or both x -values, $-\frac{5}{2}$ and $-\frac{1}{2}$ | A1 | |
| | Obtain both stationary points | A1 | |
| | | 5 | |

| Question Number | Scheme | Marks |
|-----------------|---|---------------------------------------|
| 9 (i) | $2\log_3(4x+5) - \log_3(x+3) = 2 \Rightarrow \log_3 \frac{(4x+5)^2}{(x+3)} = 2$ $\Rightarrow \frac{(4x+5)^2}{(x+3)} = 9$ $\Rightarrow 16x^2 + 31x - 2 = 0$ $(16x-1)(x+2) = 0 \Rightarrow x = \frac{1}{16} \text{ only}$ | M1, M1 A1 dM1, A1 (5) |
| (ii) (a) | <p>States that $\log a + \log b = \log(ab)$ or else uses rule and proceeds from given equation $\log a + \log b = \log(a+b)$ to $\log ab = \log(a+b)$</p> <p>Deduces $ab = a+b \Rightarrow ab - a = b \Rightarrow a(b-1) = b \Rightarrow a = \frac{b}{b-1}$ *</p> | B1 M1, A1* |
| (b) | <p>States either $b > 1$ or $b \neq 1$ as a would not be defined</p> <p>$b > 1$ as logs only exist for positive numbers</p> | B1 B1 (5) |
| | | (10 marks) |

| Question Number | Scheme | Marks |
|-----------------|--|------------------|
| 4(a) | $(2+px)^6 = (2^6 +) 6 \times 2^5 (px) + \frac{6 \times 5}{2} \times 2^4 (px)^2 + \dots$ <p>or</p> $\left(1 + \frac{p}{2}x\right)^6 = 1 + 6 \times \left(\frac{p}{2}x\right) + \frac{6 \times 5}{2} \times \left(\frac{p}{2}x\right)^2 + \dots$ | M1 |
| | 2^6 or 64 | B1 |
| | +192px or +240p ² x ² | A1 |
| | (64+)192px + 240p ² x ² | A1 |
| | | (4) |
| (b) | $\left(3 - \frac{1}{2}x\right)(2+px)^6 \Rightarrow \left(3 - \frac{1}{2}x\right)(64 + 192px + 240p^2x^2)$ | |
| | Attempts $3 \times "240p^2"$ and $\left(-\frac{1}{2}\right) \times "192p"$ | M1 |
| | $3 \times "240p^2" + \left(-\frac{1}{2}\right) \times "192p" = -\frac{3}{4}$ | dM1 |
| | $2880p^2 - 384p + 3 = 0 \Rightarrow p = \dots$ | ddM1 |
| | $(p =) \frac{1}{8}, \frac{1}{120}$ | A1 |
| | | (4) |
| | | (8 marks) |

| Question Number | Scheme | Marks |
|-----------------|--|--|
| 7.(a) | $y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44$ $y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \Rightarrow \left(\frac{dy}{dx}\right) = 3x^2 - 10x^{\frac{3}{2}} - \frac{k}{2}x^{-\frac{1}{2}} + 28$ | M1, A1 (2) |
| (b) | Subs $x=9$ into $\frac{dy}{dx}$ and sets $= 0 \Rightarrow 3 \times 81 - 10 \times 27 - \frac{k}{6} + 28 = 0$ $243 - 270 - \frac{k}{6} + 28 = 0 \Rightarrow \frac{k}{6} = 1 \Rightarrow k = 6 \quad *$ | M1 A1* (2) |
| (c) | $\int x^3 - 4x^{\frac{5}{2}} - 6x^{\frac{1}{2}} + 28x - 44 \, dx = \frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x$ Correct value of $y = -53$ at T Shaded area = "53" $\times 9 + \left[\frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x \right]_0^9$ $= 248$ | M1, A1, A1 B1 dM1 A1 (6) (10 marks) |

| Question | Answer | Marks | Guidance |
|----------|---|-------|----------------------|
| 1 | Use correct logarithm property to simplify left-hand side | M1 | Or equivalent method |
| | Use correct process to obtain equation without logarithms | M1 | |
| | Obtain $\frac{2x+1}{x-3} = e^2$ | A1 | OE |
| | Obtain $x = \frac{3e^2+1}{e^2-2}$ | A1 | OE |
| | | 4 | |

| Question Number | Please read notes for 8(i) before looking at scheme | | Marks |
|-----------------|--|--|------------|
| 8.(i) | $8^{2x+1} = 6 \Rightarrow 2x+1 = \log_8 6$ M1 $\Rightarrow 2x+1 = \frac{\log_2 6}{\log_2 8}$ A1 $\Rightarrow 2x+1 = \frac{\log_2 2 + \log_2 3}{3}$ M1 $\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$ A1 | $2^{6x+3} = 6$ $\Rightarrow (6x+3)\log_2 2 = \log_2 6$ M1 A1 $\Rightarrow (6x+3) = \log_2 2 + \log_2 3$ M1 $\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$ A1 | (4) |
| | (ii) $\log_5(7-2y) = 2\log_5(y+1) - 1$ $\log_5(7-2y) = \log_5(y+1)^2 - 1$ $\log_5(7-2y) = \log_5(y+1)^2 - \log_5 5$ $(7-2y) = \frac{(y+1)^2}{5}$ $y^2 + 12y - 34 = 0 \Rightarrow y =$ | $2\log_5(y+1) - \log_5(7-2y) = 1$ $\log_5(y+1)^2 - \log_5(7-2y) = 1$ $\log_5 \frac{(y+1)^2}{(7-2y)} = 1$ $\frac{(y+1)^2}{(7-2y)} = 5$ $y^2 + 12y - 34 = 0 \Rightarrow y =$ $y = -6 + \sqrt{70}$ oe only | |

| Question | Scheme | Marks |
|-------------|---|------------------|
| 4(a) | $\left(2 + \frac{x}{8}\right)^{13} = 8192 + \dots$ | B1 |
| | $+ \binom{13}{1} 2^{12} \left(\frac{x}{8}\right)^1 + \binom{13}{2} 2^{11} \left(\frac{x}{8}\right)^2 + \binom{13}{3} 2^{10} \left(\frac{x}{8}\right)^3 + \dots$ | M1 |
| | $\left(2 + \frac{x}{8}\right)^{13} = (8192) + 6656x + 2496x^2 + 572x^3 + \dots$ | A1A1 |
| | | (4) |
| (b) | $\frac{x}{8} = 0.0125 \Rightarrow x = 0.1$ | B1 |
| | $2.0125^{13} \approx 8192 + 665.6 + 24.96 + 0.572$ | M1 |
| | $= 8883.132$ cao | A1 |
| | | (3) |
| (c) | As all the terms in the expansion are positive, the truncated series will give an underestimate of the actual value. | B1 |
| | | (1) |
| | | (8 marks) |