

Question Number	Scheme	Marks
4(a)	$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3$ $\Rightarrow \left(\frac{dy}{dx}\right) = 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}}$	M1A1
		(2)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow 4x - 9 = 0 \Rightarrow x = \dots$	M1
	$x = \frac{9}{4} \text{ oe e.g. 2.25}$	A1
		(2)

4(c)(i)	$\left(\frac{d^2y}{dx^2}\right) = -x^{-\frac{3}{2}} + \frac{27}{4}x^{-\frac{5}{2}} \text{ oe e.g. } -x^{-\frac{3}{2}} + 6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{d^2y}{dx^2}\right)_{x=\frac{9}{4}}\right) = -\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}} \left(= \frac{16}{27}(0.5925\dots) \right)$ $\left(\frac{d^2y}{dx^2}\right) > 0 \text{ so (local) minimum}$	B1
		(2)
(d)	$0 < x < \frac{9}{4}$	B1ft
		(1)
		Total 7

Question Number	Scheme	Marks
3.(a)	<p>OPEN TOPPED</p> $120 = 3x^2y \Rightarrow y = \frac{40}{x^2}$ $A = 3x^2 + 8xy$ $= 3x^2 + 8x \times \frac{40}{x^2} = 3x^2 + \frac{320}{x}$	M1, A1
	<p>CLOSED CONTAINER</p> $120 = 3x^2y \Rightarrow y = \frac{40}{x^2}$ $A = 6x^2 + 8xy$ $= 6x^2 + 8x \times \frac{40}{x^2} = 6x^2 + \frac{320}{x}$	dM1, A1
(b)	$\frac{dA}{dx} = 6x - \frac{320}{x^2}$ $\frac{dA}{dx} = 6x - \frac{320}{x^2} = 0$ $\Rightarrow x^3 = \frac{160}{3} \Rightarrow x = \text{awrt } 3.76$	M1, A1ft
	$\frac{dA}{dx} = 12x - \frac{320}{x^2}$ $\frac{dA}{dx} = 12x - \frac{320}{x^2} = 0$ $\Rightarrow x^3 = \frac{80}{3} \Rightarrow x = \text{awrt } 2.99$	dM1, A1
(c)	<p>Attempts $\left.\frac{d^2A}{dx^2}\right _{x=3.76} = 6 + \frac{640}{x^3}$</p> $= 6 + \frac{640}{3.76^3} = \dots$	M1
	<p>Attempts $\left.\frac{d^2A}{dx^2}\right _{x=2.99} = 12 + \frac{640}{x^3}$</p> $= 12 + \frac{640}{2.99^3} = \dots$	M1
	$\left.\frac{d^2A}{dx^2}\right _{x=3.76} = \text{awrt } 18 > 0 \text{ MINIMUM*}$	A1*
	$\left.\frac{d^2A}{dx^2}\right _{x=2.99} = \text{awrt } 36 > 0 \text{ MINIMUM*}$	A1*
		(2)
		(10 marks)

Question Number	Scheme	Marks
6 (i) (a)	Uses $\sin^2 \theta + \cos^2 \theta = 1$ with $\cos \theta = \frac{1}{\sqrt{5}} \Rightarrow \sin^2 \theta = \frac{4}{5}$	M1
	$\Rightarrow \sin \theta = -\frac{2}{\sqrt{5}}$	A1
(b)	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ with $\cos \theta = \frac{1}{\sqrt{5}}$ and their $\sin \theta = -\frac{2}{\sqrt{5}}$	M1
	$\Rightarrow \tan \theta = -2$	A1
(ii) (a)	1 (m)	B1
(b)	$30t - 40 = 180 \Rightarrow t = 7.33$ Hence 7:20 am	M1, A1, A1
(c)	$4 + 3 \cos(30T - 40)^\circ = 3.5 \Rightarrow \cos(30T - 40)^\circ = -\frac{1}{6}$ $\Rightarrow 30T - 40 = 99.6, 260.4, \underline{459.6}$ $\Rightarrow T = \frac{"459.6"+40}{30} = 16.65$	M1 A1 dM1, A1
		(4) (1) (3) (4) (12 marks)

Question Number	Scheme	Marks
3(a)	Attempts $(8-3)^2 + (5--7)^2 = \dots$	M1
	Writes $(x-3)^2 + (y-5)^2 = k$	M1
	$(x-3)^2 + (y-5)^2 = 169$	A1
		(3)
(b)	Attempts $d^2 + (2\sqrt{22})^2 = 169 \Rightarrow d = \dots$	M1
	States or uses $(y=) 5 + d$	dM1
	$y = 14$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
6(a)	$\frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \tan \theta \Rightarrow \frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \frac{\sin \theta}{\cos \theta}$	<u>M1</u>
	$\frac{3 \sin \theta \cos \theta}{2 \sin \theta - 1} = 5 \frac{\sin \theta}{\cos \theta} \Rightarrow 3 \sin \theta \cos^2 \theta = 5 \sin \theta (2 \sin \theta - 1)$	M1
	$3 \sin \theta \cos^2 \theta = 10 \sin^2 \theta - 5 \sin \theta \Rightarrow 3 \sin \theta (1 - \sin^2 \theta) = 10 \sin^2 \theta - 5 \sin \theta *$	M1
	$\Rightarrow 3 \sin^3 \theta + 10 \sin^2 \theta - 8 \sin \theta = 0$	A1*
		(4)
(b)	$3 \sin^3 2x + 10 \sin^2 2x - 8 \sin 2x = 0 \Rightarrow \sin 2x (3 \sin^2 2x + 10 \sin 2x - 8) = 0$	M1
	$(3 \sin 2x - 2)(\sin 2x + 4) = 0 \Rightarrow \sin 2x = \dots$	
	$\sin 2x = \frac{2}{3}$	A1
	$(x =) 0.365$	A1
	$x = 0$	B1
	(4)	
		(8 marks)

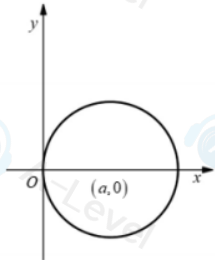
Question	Answer	Marks	Guidance
8(a)	Use $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\sec \theta = \frac{1}{\cos \theta}$ to obtain $6 \sin \theta$	B1	
	Expand second term to obtain $5\sqrt{3} \cos \theta + 5 \sin \theta$	B1	
	Simplify to obtain $11 \sin \theta + 5\sqrt{3} \cos \theta$	B1	
	State $R = 14$	B1 FT	FT their $k_1 \cos \theta + k_2 \sin \theta$
	Use appropriate trigonometry to find α	M1	
	Obtain $\alpha = 38.21$	A1	AWRT
		6	
8(b)	State or imply $14 \sin(2\beta + 38.21) = 2$	B1 FT	FT their R and α
	Carry out correct process to find value of β between 0° and 90°	M1	
	Obtain 66.8	A1	AWRT
			3

Question	Answer	Marks	Guidance
5(a)	Carry out division at least as far as $x^2 + kx$ or equivalent ...	M1	OE, e.g. comparing coefficients with coefficient of x^2 equal to 1 and attempt at a second coefficient.
	Obtain quotient $x^2 + 4x + 12$	A1	
	Confirm remainder is 7	A1	AG
		3	
5(b)	Include $(x-2)^2$ as a factor	M1	Must be a product of factors only SC B1 for $(x^2 - 4x + 4)(x^2 + 4x + 12)$
	Conclude $(x-2)^2(x^2 + 4x + 12)$	A1	isw any attempt to factorise the quotient.
		2	
5(c)	Apply logarithms and use power law for $e^{-3y} = k$ where $k > 0$	M1	
	Obtain $y = -\frac{1}{3}\ln 2, \frac{1}{3}\ln \frac{1}{2}$	A1	Or exact equivalent Must be simplified e.g. not $\ln e$ or $\frac{6}{3}$ ISW extra solutions but A0 if undefined solutions are included.
		2	

Question Number	Scheme	Marks
3 (a)	$f(x) = (3x^2 - 4x - 5)(x - k) - 5$	B1 (1)
	States -5	
(b)	Sets $f(-2) = 25 \rightarrow (3 \times 4 - 4 \times -2 - 5)(-2 - k) - 5 = 25$	M1 A1* (2)
	$15(-2 - k) = 30 \Rightarrow -2 - k = 2 \Rightarrow k = -4$ *	
(c)	$(3x^2 - 4x - 5)(x + 4) - 5 = 3x^3 + 8x^2 - 21x - 25$	B1 M1 A1ft A1 (4) (7 marks)
	Attempts $3x-1 \overline{)3x^3 + 8x^2 - 21x - 25}$ to achieve quotient of $\dots x^2 + \dots x + \dots$	
	and a remainder that is a constant	
	Quotient = $x^2 + 3x - 6$ OR Remainder = -31	
	Quotient = $x^2 + 3x - 6$ AND Remainder = -31	

Question Number	Scheme	Marks
7.(a)	$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44$ $y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \Rightarrow \left(\frac{dy}{dx}\right) = 3x^2 - 10x^{\frac{3}{2}} - \frac{k}{2}x^{-\frac{1}{2}} + 28$	M1, A1 (2)
(b)	Subs $x=9$ into $\frac{dy}{dx}$ and sets $= 0 \Rightarrow 3 \times 81 - 10 \times 27 - \frac{k}{6} + 28 = 0$ $243 - 270 - \frac{k}{6} + 28 = 0 \Rightarrow \frac{k}{6} = 1 \Rightarrow k = 6 \quad *$	M1 A1* (2)
(c)	$\int x^3 - 4x^{\frac{5}{2}} - 6x^{\frac{1}{2}} + 28x - 44 \, dx = \frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x$ Correct value of $y = -53$ at T Shaded area = "53" $\times 9 + \left[\frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x \right]_0^9$ $= 248$	M1, A1, A1 B1 dM1 A1 (6)
		(10 marks)

Question Number	Scheme	Marks
9(a)	$(0, 5)$	B1 (1)
(b)	$x^2 - 4x + 5 = 2 \Rightarrow x^2 - 4x + 3 = 0$ $\Rightarrow x = \dots$	M1
	$x(E) = 1, x(F) = 3$	A1 (2)
(c)	$\text{Area } R_1 = \int_0^1 (x^2 - 4x + 5 - 2) \, dx = \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 = \frac{1}{3} - 2 + 3 - 0 = \frac{4}{3}$	M1A1
	$\text{Area } R_2 = \frac{1}{2} \times 3 \times 3 - \frac{4}{3} = \frac{19}{6}$ or $\int_0^1 (5 - x - (x^2 - 4x + 5)) \, dx + \frac{1}{2} \times 2 \times 2 = \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^1 + 2 = \frac{3}{2} - \frac{1}{3} + 2 = \frac{7}{6} + 2 = \frac{19}{6}$ or $\frac{1}{2} \times 3 \times 3 - \int_0^1 (x^2 - 4x + 5 - 2) \, dx = \frac{9}{2} - \left[\frac{x^3}{3} - 2x^2 + 3x \right]_0^1 = \frac{9}{2} - \left(\left(\frac{1}{3} - 2 + 3 \right) - 0 \right) = \frac{19}{6}$	M1A1
	$\frac{\text{area of } R_1}{\text{area of } R_2} = \frac{\frac{4}{3}}{\frac{19}{6}} = \frac{8}{19}$	A1 (5)
		Total 8

Question Number	Scheme	Marks
9(a)		B1 (1)
(b)	$(x \pm a)^2 + y^2 = \dots$ $(x \pm a)^2 + y^2 = a^2$ <p>Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ to form and solve an equation in a</p> <p>E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow 10a = 61 \Rightarrow a = 6.1$</p> $(x - 6.1)^2 + y^2 = 6.1^2$	M1 A1 dM1 A1 (4) (5 marks)

Question Number	Scheme	Marks
1(a)	$(u_2 =) k - \frac{8}{1}$ and $(u_3 =) k - \frac{8}{k-8}$ $\left(= \frac{k^2 - 8k - 8}{k-8} \right)$ oe	M1A1 (2)
(b)	$u_3 = 6 \Rightarrow k - \frac{8}{k-8} = 6 \Rightarrow k^2 - 14k + 40 = 0$ $(k - 4)(k - 10) = 0 \Rightarrow k = \dots$ or $(k =) \frac{14 \pm \sqrt{14^2 - 4 \times 1 \times 40}}{2 \times 1}$ $(k =) 4, 10$	M1 dM1 A1 (3)
		(5 marks)

Question Number	Scheme	Marks
5(a)	Identifies $h = 1.5$ $\text{Area} = \frac{1.5}{2} \{12 + 0.023 + 2(4.243 + 1.5 + 0.530 + 0.188 + 0.066)\}$ $= 18.8$	B1 M1 A1 (3)
(b) (i)	$\int_{-2}^7 3 \left(\frac{1}{2} \right)^{x+2} dx = \frac{1}{4} \int_{-2}^7 3 \left(\frac{1}{2} \right)^x dx = \frac{1}{4} \times "18.8" = 4.7$	B1ft
(ii)	$\int_{-2}^7 (2^{-x} + 2x) dx = \frac{1}{3} \times "18.8" + [x^2]_{-2}^7 = 51.3$	M1, A1ft (3) (6 marks)