Question Number	Scheme	Marks
	$y=\frac{2}{3}x^2-9\sqrt{x}+13$	
9 (a)	$\frac{dy}{dx}=\frac{4}{3}x-\frac{9}{2\sqrt{x}}$	M1, A1
	Attempts to solve $\frac{dy}{dx}>0\Rightarrow x^{\frac{3}{2}}>\frac{27}{8}\Rightarrow x>\frac{9}{4}$	dM1, A1
		(4)
(b)	$\int\frac{2}{3}x^2-9\sqrt{x}+13\,dx=\frac{2}{9}x^3-6x^{\frac{3}{2}}+13x\quad(+c)$	M1
	Area = $\left[\frac{2}{9}x^3-6x^{\frac{3}{2}}+13x\right]_0^9=117$	dM1, A1
	Substitutes $x=9$ into $\frac{dy}{dx}=\frac{4}{3}x-\frac{9}{2\sqrt{x}}\Rightarrow\frac{dy}{dx}=\left(\frac{21}{2}\right)$	M1
	Finds where tangent meets x-axis $0-40=\frac{21}{2}(x-9)\Rightarrow x=\frac{109}{21}$	dM1 A1
	Area of R = $\left[\frac{2}{9}x^3-6x^{\frac{3}{2}}+13x\right]_0^9-\frac{1}{2}\times40\times\left(9-\frac{109}{21}\right)$	ddM1
	$=117-\frac{1600}{21}=\frac{857}{21}$	A1
		(8) **(12 marks)**

Question Number	Scheme	Marks
8(a)	$x^2+3=13-\frac{9}{x^2}\Rightarrow x^4+3x^2=13x^2-9$ $\Rightarrow x^4-10x^2+9=0$	M1A1
	$x^4-10x^2+9=0\Rightarrow(x^2-1)(x^2-9)=0\Rightarrow x^2=\ldots$ $\Rightarrow x=\ldots$	M1
	$x=1,\quad x=3$	A1
		(4)
(b)	$\int\left\{13-\frac{9}{x^2}-(x^2+3)\right\}dx=13x+\frac{9}{x}-\frac{x^3}{3}-3x(+c)$ or $\int\left(13-\frac{9}{x^2}\right)dx=13x+\frac{9}{x}(+c),\quad\int(x^2+3)dx=\frac{x^3}{3}+3x(+c)$	M1A1
	$\left[10x+\frac{9}{x}-\frac{x^3}{3}\right]_1^3=10(3)+\frac{9}{3}-\frac{3^3}{3}-\left(10+9-\frac{1}{3}\right)=\ldots$ or $\left[13x+\frac{9}{x}\right]_1^3-\left[\frac{x^3}{3}+3x\right]_1^3=39+3-(13+9)-\left\{9+9-\left(\frac{1}{3}+3\right)\right\}=\ldots$	**dM1**
	$=\frac{16}{3}$	A1
		(4)
		Total 8

Question	Scheme	Marks
10(a)	Equation of circle is $(x-3)^2+(y-5)^2=r^2$ and line is $y=2x+k$ So intersect when $(x-3)^2+(2x+k-5)^2=r^2$	M1
	$\Rightarrow x^2-6x+9+4x^2+4(k-5)x+(k-5)^2=r^2$ $\Rightarrow 5x^2+(-6+4k-20)x+9+k^2-10k+25-r^2=0$	dM1
	$\Rightarrow 5x^2+(4k-26)x+k^2-10k+34-r^2=0$ *	A1*
		(3)
(b)	Tangent to $C \Rightarrow b^2-4ac=0 \Rightarrow (4k-26)^2-4\times5\times(k^2-10k+34-r^2)=0$	M1
	$\Rightarrow 16k^2-208k+676-20k^2+200k-680+20r^2=0$ $\Rightarrow 5r^2=\ldots$	M1
	$\Rightarrow 5r^2=k^2+2k+1=(k+1)^2$	A1
		(3)
(b) Way 2	Gradient of BX is $-\frac{1}{2}$ so equation of BX is $y-5=-\frac{1}{2}(x-3)$ $y-5=-\frac{1}{2}(x-3), y=2x+k \Rightarrow x=\ldots, y=\ldots\left(\frac{13-2k}{5},\frac{26+k}{5}\right)$	M1
	$\left(\frac{13-2k}{5}-3\right)^2+\left(\frac{26+k}{5}-5\right)^2=r^2$	dM1
	$\Rightarrow 5r^2=k^2+2k+1=(k+1)^2$	A1
		(3)
(c)	Triangle AXB is right angled so $AB^2+r^2=XA^2=(3-0)^2+(5-k)^2$	M1
	$AB^2=4r^2$ so $AB^2+r^2=5r^2$	M1
	$\Rightarrow 5r^2=9+(5-k)^2$	A1
	$\Rightarrow k^2+2k+1=9+25-10k+k^2$	M1
	$\Rightarrow 12k=33 \Rightarrow k=\ldots$	dM1
	$k=\frac{11}{4}$	A1
		(6)
		(12 marks)

Question Number	Scheme	Marks
8 (a)	$y=\frac{4}{3}x^3-11x^2+kx \Rightarrow \frac{dy}{dx}=4x^2-22x+k$	M1
	uses $x=2, \frac{dy}{dx}=0 \Rightarrow 0=16-44+k \Rightarrow k=28$ *	dM1 A1*
		(3)
(b)	$\frac{dy}{dx}=4x^2-22x+28=0 \Rightarrow (2x-4)(2x-7)=0 \Rightarrow x=...$	M1
	$x<2, x>\frac{7}{2}$	A1
		(2)
(c)	$\int\left(\frac{4}{3}x^3-11x^2+28x\right)dx \Rightarrow \frac{1}{3}x^4-\frac{11}{3}x^3+14x^2$	M1 A1
	Correct y coordinate of $M=\frac{68}{3}$	B1
	Complete method to find $R = 2\times"\frac{68}{3}"-\int_0^2\left(\frac{4}{3}x^3-11x^2+28x\right)dx$	M1
	$=2\times"\frac{68}{3}"-\left(\frac{1}{3}\times2^4-\frac{11}{3}\times2^3+14\times2^2\right)$	
	$=\frac{40}{3}$	A1
		(5) **(10 marks)**

Question Number	Scheme	Marks
8(a)	$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$ $\Rightarrow x^4-10x^2+9=0$	M1A1
	$x^4-10x^2+9=0 \Rightarrow (x^2-1)(x^2-9)=0 \Rightarrow x^2=...$ $\Rightarrow x=...$	M1
	$x=1, \quad x=3$	A1
		(4)
(b)	$\int\left\{13-\frac{9}{x^2}-(x^2+3)\right\}dx=13x+\frac{9}{x}-\frac{x^3}{3}-3x(+c)$ or $\int\left(13-\frac{9}{x^2}\right)dx=13x+\frac{9}{x}(+c), \quad \int(x^2+3)dx=\frac{x^3}{3}+3x(+c)$	M1A1
	$\left[10x+\frac{9}{x}-\frac{x^3}{3}\right]_1^3=10(3)+\frac{9}{3}-\frac{3^3}{3}-\left(10+9-\frac{1}{3}\right)=...$ or $\left[13x+\frac{9}{x}\right]_1^3-\left[\frac{x^3}{3}+3x\right]_1^3=39+3-(13+9)-\left\{9+9-\left(\frac{1}{3}+3\right)\right\}=...$	**dM1**
	$=\frac{16}{3}$	A1
		(4)
		Total 8

Question Number	Scheme	Marks
9(a)	$y = x^3 - 5x^2 + 3x + 14 \Rightarrow \frac{dy}{dx} = 3x^2 - 10x + 3 = 0$	M1
	Roots are $3, \frac{1}{3}$ $\Rightarrow$ when $x = 3,\ y = "3"^3 - 5 \times "3"^2 + 3 \times "3" + 14 = ...$	dM1
	Centre is $(3, 5)$	A1
		(3)
(b)	At A $\quad y = 8$	B1
	$r^2 = (2 - "3")^2 + ("8" - "5")^2 \quad (=10)$	M1
	$(x-3)^2 + (y-5)^2 = 10$	A1
		(3)
(c)	$\frac{"8" - "5"}{2 - "3"} = ...(-3)$	M1
	$y - "8" = "\frac{1}{3}"(x - 2)$	M1
	$y = \frac{1}{3}x + \frac{22}{3}$ $\quad$ *	A1*
		(3)
(d)	$\int_0^2 x^3 - 5x^2 + 3x + 14 \, dx = ...\left(\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x\right)$	M1
	Area $= \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 14x\right]_0^2 - \left(\frac{1}{2} \times \left(\frac{22}{3} + "8"\right) \times 2\right) = ...$ $\frac{1}{4} \times 16 - \frac{5}{3} \times 8 + \frac{3}{2} \times 4 + 14 \times 2 - \frac{46}{3}$	dM1
	$\frac{74}{3} - \frac{46}{3} = \frac{28}{3}$	A1
		(3)
		(12 marks)

Question	Scheme	Marks
6(a)	Way 1: Eqn is $(x-3)^2+(y+4)^2-9-16+k=0$	M1
	So radius must be 4 $\Rightarrow 25-k=16$	M1
	$\Rightarrow k=9$	A1
		(3)
	Way 2: $y=0 \Rightarrow x^2-6x+k=0$ has one solution	M1
	$\Rightarrow 6^2-4\times1\times k=0$	M1
	$\Rightarrow k=9$	A1
		(3)
(b)	C intersects x axis at (3,0)	B1
	Intersects y axis when $y^2+8y+9=0 \Rightarrow y=...$ or $y="-4"\pm\sqrt{"16"-"9"}$ Or uses base of triangle is $2\sqrt{"16"-"9"}$	M1
	Area $RST=\dfrac{1}{2}\times2\sqrt{7}\times3$	M1
	$=3\sqrt{7}$	A1
		(4)
		(7 marks)

Question Number	Scheme	Notes	Marks
9(a)	$mx = x - x^2 \Rightarrow m = 1 - x \Rightarrow x = ...$ Or $y = \frac{y}{m} - \frac{y^2}{m^2} \Rightarrow m^2 = m - y \Rightarrow y = ...$	Attempts to eliminate either *x* or *y* and factors out or cancels *x*/*y* to get a linear equation and solve.	M1
	$x = 1 - m$ **and** $y = m(1-m)$	Both correct	A1
			(2)
(b)	$\int x - x^2(-mx)\,dx = \frac{x^2}{2} - \frac{x^3}{3}\left(-m\frac{x^2}{2}\right)$	$x^n \to x^{n+1}$ in at least one term	M1
	Area of $R_1 = \int_0^{"1-m"} \{x - x^2(-mx)\}\,dx$ $= \frac{(1-m)^2}{2} - \frac{(1-m)^3}{3}\left(-m\frac{(1-m)^2}{2}\right) - 0$	Uses the limits "1 – *m*" and 0 in their integrated expression and subtracts (condone the omission of the "– 0")	M1
		Correct expression in *m* with/without the area under line subtracted.	A1
	Area of $R_1 = \int_0^{"1-m"} \{x - x^2 - mx\}\,dx = \frac{(1-m)^2}{2}(1-m) - \frac{(1-m)^3}{3}(-0)$ Correct strategy for the area (may be scored for finding separate areas and subtracting)		dM1
	$= \frac{(1-m)^3}{6}$ *	Correct expression	A1*
			(5)
(c)	Area of $(R_1 + R_2) = \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = ...$ $\left(= \frac{1}{6}\right)$	Correct method for finding the area of $R_1 + R_2$ Alternatively, a correct method for finding the area of R_2	M1
	Alt: Area of $R_2 = \int_{1-m}^1 (x - x^2)\,dx + \frac{1}{2}("1-m") \times m(1-m)$ $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{1-m}^1 + \frac{1}{2}m(1-m)^2 = ...\left(= \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2}m(1-m)^2\right)$		
	$R_1 = R_2 \Rightarrow \frac{(1-m)^3}{6} = \frac{1}{12} \Rightarrow m = ...$	Sets up a correct equation using the answer to part (b) and solves for *m*	dM1
	Alt: $\frac{(1-m)^3}{6} = \frac{1}{6} - \frac{(1-m)^2}{2} + \frac{(1-m)^3}{3} + \frac{1}{2}m(1-m)^2 \Rightarrow m = ...$		
	$m = 1 - \frac{1}{\sqrt[3]{2}}$	Correct exact value in any form	A1
			(3)
			Total 10

Question Number	Scheme	Marks
6 (a)	Centre of circle is midpoint of $(-2,18)$ and $(14,6)=(6,12)$	B1
	Attempts radius² or diameter². E.g. $D^2=(14--2)^2+(6-18)^2=400$	M1
	Radius² = 100	A1
	$(x-6)^2+(y-12)^2=100$	M1, A1
		(5)
(b)	Recognises equation of C_2 is $x^2+y^2=k^2$	B1
	Attempts to find one value of k or k^2 Look for $\sqrt{"6"^2+"12"^2}\pm\sqrt{"100"}$	M1
	$x^2+y^2=\left(6\sqrt{5}+10\right)^2$ or $x^2+y^2=\left(6\sqrt{5}-10\right)^2$ o.e.	A1
	$x^2+y^2=\left(6\sqrt{5}+10\right)^2$ and $x^2+y^2=\left(6\sqrt{5}-10\right)^2$ o.e.	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
	$y=x^3-4x^{\frac{5}{2}}-kx^{\frac{1}{2}}+28x-44$	
7.(a)	$y=x^3-4x^{\frac{5}{2}}-kx^{\frac{1}{2}}+28x-44\Rightarrow\left(\frac{dy}{dx}\right)=3x^2-10x^{\frac{3}{2}}-\frac{k}{2}x^{-\frac{1}{2}}+28$	M1, A1
		(2)
(b)	Subs $x=9$ into $\frac{dy}{dx}$ and sets = 0 $\Rightarrow 3\times81-10\times27-\frac{k}{6}+28=0$	M1
	$243-270-\frac{k}{6}+28=0\Rightarrow\frac{k}{6}=1\Rightarrow k=6$ *	A1*
		(2)
(c)	$\int x^3-4x^{\frac{5}{2}}-6x^{\frac{1}{2}}+28x-44\,dx=\frac{1}{4}x^4-\frac{8}{7}x^{\frac{7}{2}}-4x^{\frac{3}{2}}+14x^2-44x$	M1, A1, A1
	Correct value of $y=-53$ at T	B1
	Shaded area = $"53"\times9+\left[\frac{1}{4}x^4-\frac{8}{7}x^{\frac{7}{2}}-4x^{\frac{3}{2}}+14x^2-44x\right]_0^9$	dM1
	$=248$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
9(a) (i)	Centre $= (k, 2k)$	B1
(ii)	Radius $= \sqrt{k+7}$	B1
		(2)
(b)(i)	$(2-k)^2 + (3-2k)^2 = k+7 \Rightarrow 4-4k+k^2+9-12k+4k^2 = k+7$	M1
	$5k^2 - 17k + 6 = 0$ *	A1*
(ii)	$(k=)\dfrac{2}{5}, 3$	B1
		(3)
(c)		
	Centre is $\left(\dfrac{2}{5}, \dfrac{4}{5}\right)$	B1ft
	Gradient of tangent $\pm\dfrac{2-"\frac{2}{5}"}{3-"\frac{4}{5}"} = \left(-\dfrac{8}{11}\right)$	M1
	$y-3 = "-\dfrac{8}{11}"(x-2) \Rightarrow$ sets $y=0 \Rightarrow x = ...$ Alternatively, $\tan\angle PTO = \dfrac{8}{11} \Rightarrow XT = \dfrac{3}{8/11} = ...$ where X is (2,0)	M1
	Area $OPT = \dfrac{1}{2} \times "\dfrac{49}{8}" \times 3 = ...$ Alternatively, Area $OPT = \dfrac{1}{2} \times 2 \times 3 + \dfrac{1}{2} \times "\dfrac{33}{8}" \times 3 = ...$	dM1
	$= \dfrac{147}{16}$ oe	A1
		(5)
		(10 marks)

Question	Answer	Marks	Guidance
6(a)	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1	
	Obtain $\sin\theta = \frac{1}{6}$	B1	
		2	
6(b)	Use correct identity or identities to find value of $\sec\theta$	M1	
	Obtain $\frac{6}{\sqrt{35}}$ or exact equivalent	A1	
		2	
6(c)	Use correct identity or identities to find value of $\cos 2\theta$	M1	
	Obtain $\frac{17}{18}$ or exact equivalent	A1	
		2	

(b)	$x^n \to x^{n+1}$	For increasing any power of x by 1 for C_1 or C_2 or for $\pm(C_1 - C_2)$	M1
	$\pm\int\{-2x^2+7x-1-(x^3-6x+9)\}\mathrm{d}x = \pm\int(-x^3-2x^2+13x-10)\mathrm{d}x$ $= \pm\left(-\frac{x^4}{4}-\frac{2x^3}{3}+\frac{13x^2}{2}-10x\right)$ or $\pm\left\{\int(-2x^2+7x-1)\mathrm{d}x - \int(x^3-6x+9)\mathrm{d}x\right\}$ $= \pm\left(-\frac{2x^3}{3}+\frac{7x^2}{2}-x-\left(\frac{x^4}{4}-\frac{6x^2}{2}+9x\right)\right)$ or $\int(-2x^2+7x-1)\mathrm{d}x = -\frac{2x^3}{3}+\frac{7x^2}{2}-x,\ \int(x^3-6x+9)\mathrm{d}x = \frac{x^4}{4}-\frac{6x^2}{2}+9x$ dM1: For correct integration of 1 term for C_1 **and** one term for C_2 or for correct integration for 2 terms of <u>their</u> $\pm(C_1 - C_2)$ A1: Fully correct integration of both C_1 and C_2 or for $\pm(C_1 - C_2)$. **Award this mark as soon as fully correct integration is seen and ignore subsequent work.**		dM1A1
	$= -\frac{2^4}{4}-\frac{2(2)^3}{3}+\frac{13(2)^2}{2}-10(2)-\left(-\frac{1^4}{4}-\frac{2(1)^3}{3}+\frac{13(1)^2}{2}-10(1)\right)$ Fully correct strategy for the area. Depends on both previous M marks. Uses the limits "2" and 1 in their "changed" expression(s) and subtracts either way round.		ddM1
	$= \frac{13}{12}$ If the attempt is correct apart from subtracting the wrong way round (for limits or functions) and $-\frac{13}{12}$ is obtained, allow recovery if they then make their answer positive.		A1
			(5)
			Total 9

Question Number	Scheme	Marks
10 (a)	$y=2x+\frac{64}{x^2}-3$	
	$\frac{dy}{dx}=2-\frac{128}{x^3}$	M1, A1
	Attempts to solve $\frac{dy}{dx}\Rightarrow x^3=64\Rightarrow x=4$ *	dM1, A1*
		(4)
(b)	$\int 2x+\frac{64}{x^2}-3\,dx=x^2-\frac{64}{x}-3x\quad(+c)$	M1, A1
	Finds the y values at both $x=2$ and $x=4$. $M=(0,\,17)$ and $N=(0,\,9)$	B1
	Full attempt at area = $\left[x^2-\frac{64}{x}-3x\right]_2^4-9\times2+2\times(17-9)=22-18+16=20$	dM1, A1
		(5)
		(9 marks)