Question Number	Scheme	Marks
11(i)	E.g. $n=5 \Rightarrow 3^5+2=245$	M1
	245 is not a prime number or e.g. 245 is divisible by 5 so not true	A1
		(2)

(ii)	$m=3k+1 \Rightarrow m^2-1=(3k+1)^2-1=9k^2+6k+1-1$ **or** $m=3k+2 \Rightarrow m^2-1=(3k+2)^2-1=9k^2+12k+4-1$	M1
	$m^2-1=9k^2+6k=3(3k^2+2k)$ **or** $m^2-1=9k^2+12k+3=3(3k^2+4k+1)$	A1
	$m=3k+1 \Rightarrow m^2-1=(3k+1)^2-1=9k^2+6k+1-1$ **and** $m=3k+2 \Rightarrow m^2-1=(3k+2)^2-1=9k^2+12k+4-1$	**d**M1
	$3(3k^2+2k)$ and $3(3k^2+4k+1)$ are both multiples of 3 so m^2-1 must be divisible by 3 when m is not divisible by 3	A1
		(4)
		Total 6

Question Number	Scheme	Marks
10	Considers another relevant case for n e.g. $n=3k-1$ or $3k+1$ or $3k+2$ For example $n = 3k + 1$ $\Rightarrow 2n^2+n+1=2(3k+1)^2+(3k+1)+1=18k^2+15k+4$	M1
	$=3(6k^2+5k+1)+1$ which is not divisible by 3	A1
	Considers a third relevant case to complete the 3 cases for n e.g. one of $n=3k-1$ or $3k+1$ or $3k+2$ For example $n = 3k - 1$ $\Rightarrow 2n^2+n+1=2(3k-1)^2+(3k-1)+1=18k^2-9k+2$	dM1
	$=3(6k^2-3k)+2$ which is not divisible by 3 Hence $2n^2+n+1$ is not divisible by 3	A1
		(4)
		Total 4

Question Number	Scheme	Marks
4 (a)(i)	e.g. $f(-3) = 13-9-(k-3)^2 = 4-(k-3)^2$ and states when $k=5$, $f(-3)=0$	B1*
(a)(ii)	Uses $f(-3)=0 \Rightarrow (k-3)^2 = 4 \Rightarrow k = (5), 1$	M1A1 **(3)**
(b)(i)	$13+3x+(x+2)(x+5)^2 = 13+3x+(x+2)(x^2+10x+25) = 13+3x+x^3+....$	M1
	$= x^3+12x^2+48x+63$	A1
	Hence $f(x) = (x+3)(x^2+9x+21)$	dM1A1 **(4)**
(b)(ii)	e.g. Attempts "b^2-4ac" for their $(x^2+9x+21)$	M1
	e.g. $b^2-4ac=-3<0 \Rightarrow (x^2+9x+21)$ has no roots and hence $f(x)=0$ has one solution, -3	A1 **(2)** **(9 marks)**

Question Number	Scheme	Marks
5(i)	(As $x \geqslant 0$ so $\sqrt{3x}$ exists and) $\left(\sqrt{3x}-1\right)^2 \geqslant 0$	M1
	Hence $3x-2\sqrt{3x}+1 \geqslant 0$	M1
	$\Rightarrow 3x+1 \geqslant 2\sqrt{3x}$ *	A1*
		(3)
Alt 1	$3x+1 \geqslant 2\sqrt{3x} \Leftrightarrow (3x+1)^2 \geqslant 12x \Leftrightarrow 9x^2-6x+1 \geqslant 0$	M1
	$9x^2-6x+1 \geqslant 0 \Leftrightarrow (3x-1)^2 \geqslant 0$	M1
	Square numbers are greater than or equal to zero so $(3x-1)^2 \geq 0$ is true hence $3x+1 \geqslant 2\sqrt{3x}$ *	A1*
		(3)
Alt 2	If $3x+1 < 2\sqrt{3x}$ then $3x-2\sqrt{3x}+1 < 0$	M1
	So $(...\sqrt{3x} \pm ...)^2 < 0$ or $(\sqrt{x} \pm ...)^2 < 0$	M1
	But $(\sqrt{3x}-1)^2 \geq 0$ for all $x \geqslant 0$ so $3x+1 \geqslant 2\sqrt{3x}$	A1
(ii)	Shows that it is not true for three consecutive prime numbers Eg $5+7+11=23$ which is not divisible by 5 (so not true)	B1
		(1)
		(4 marks)

Question Number	Scheme	Marks
1	$f(x) = ax^3+3x^2-8x+2$	
	Sets $f(2)=3 \Rightarrow 8a+12-16+2=3$	M1
	$\Rightarrow 8a=5 \Rightarrow a=\frac{5}{8}$	M1A1
		(3)
		(3 marks)

Question Number	Scheme	Marks
4 (a)	$f(x)=4x^3+13x^2-10x+8$ $x-2\overline{)4x^3+13x^2-10x+8}$ $\quad 4x^2+21x+32$ $4x^3-8x^2$ $21x^2-10x$ $21x^2-42x$ $32x+8$ $32x-64$ 72 Synthetic Division $2 \mid 4 \quad 13 \quad -10 \quad 8$ $\quad\quad 8 \quad 42 \quad 64$ $4 \quad 21 \quad 32 \mid 72$	
(i)	$Q(x)=4x^2+21x+32$	M1, A1
(ii)	$R=72$	M1, A1 **(4)**
(b)(i)	Attempts $f(-4)=4\times-64+13\times16-10\times-4+8$ $=-256+208+40+8=0$ Hence $(x+4)$ is a factor *	M1 A1*
(ii)	$f(x)=4x^3+13x^2-10x+8=(x+4)(4x^2-3x+2)$	M1
	For their $4x^2-3x+2$ attempts "b^2-4ac" $=9-32$, $b^2-4ac'<0$ so $4x^2-3x+2$ has no real roots and $f(x)=0$ has one at $x=-4$	M1 A1* **(5)**
(c)	$(f'(x))=12x^2+26x-10=2(3x-1)(2x+5)$ $-\frac{5}{2}<x<\frac{1}{3}$	M1 dM1, A1 **(3)** **(12 marks)**

Question Number	Scheme	Marks
	$f(x)=ax^3+bx^2+5x-3$	
4(a)	Sets $f(-3)=0 \rightarrow a\times(-3)^3+b\times(-3)^2+5\times(-3)-3=0$	M1
	$\Rightarrow -27a+9b=18 \Rightarrow b=3a+2$ *	A1*
		(2)
(b)	Sets $f\left(\frac{1}{2}\right)=\frac{7}{4} \rightarrow a\times\left(\frac{1}{2}\right)^3+b\times\left(\frac{1}{2}\right)^2+5\times\left(\frac{1}{2}\right)-3=\frac{7}{4}$	M1
	$a+2b=18$	A1
	Solves $b=3a+2$ with their $a+2b=18 \Rightarrow a=\ldots, b=\ldots$	M1
	$a=2, b=8$	A1
		(4)
(c)	e.g. $\frac{2x^3+8x^2+5x-3}{x-2}=\left(\ldots x^2+\ldots x+\ldots\right)+\text{Remainder}$	M1
	(Quotient =) $2x^2+12x+29$	A1
	Remainder = 55	B1
		(3)
		(9 marks)

Question Number	Scheme					Marks
1.						
	a	*b*	*c*	*abc*	One correct set *a*, *b* & *c*	B1
	2	**15**	**3**	**90**		
	3	**9**	**8**	**216**	Two correct rows	M1
	4	**1**	**15**	**60**		
	All values of *abc* are multiples of 6				Fully correct + statement	A1
						(3 marks)

Question Number	Scheme	Marks
5(a)	$3^3+3^2(p+3)-3+q=0$	M1
	eg $\Rightarrow 27+9p+27-3+q=0\Rightarrow 9p+q=-51$*	A1*
		(2)
(b)	$(-p)^3+(p+3)(-p)^2-(-p)+q=9$	M1
	eg $-p^3+p^3+3p^2+p+q=9\Rightarrow 3p^2+p+q-9=0$*	A1*
		(2)
(c)	$3p^2+p+q-9=0$ $\Rightarrow 3p^2+p-51-9p-9=0$ $\Rightarrow 3p^2-8p-60=0$	M1
	$p=6$	A1
	$q=-51-9p=-105$	A1
		(3)
(d)	$f(x)=x^3+9x^2-x-105$ $f(x)=(x-3)(...x^2+...x+...)$	M1
	$g(x)=x^2+12x+35$	A1
		(2)
		Total 9

Question Number	Scheme	Marks
6.(a)	Sets $f(3)=0\rightarrow$ equation in k Eg. $27k-135-96-12=0$	M1
	$\Rightarrow 27k=243\Rightarrow k=9$ * (= 0 must be seen)	A1*
		(2)
(b)	$9x^3-15x^2-32x-12=(x-3)(9x^2+12x+4)$	M1 A1
	$=(x-3)(3x+2)^2$	dM1 A1
		(4)
(c)	Attempts $\cos\theta=-\frac{2}{3}$	M1
	$\theta=131.8°, 228.2°$ (awrt)	A1
		(2)
		(8 marks)

Question Number	Scheme	Marks
9.(a)	(If x and y are positive) $(\sqrt{x}-\sqrt{y})^2 \ldots 0 \Rightarrow x-\ldots\sqrt{xy}+y \ldots 0$	M1
	$\Rightarrow x-2\sqrt{xy}+y \ldots 0$	A1
	$\Rightarrow \frac{x+y}{2}\ldots\sqrt{xy}$	A1*
		(3)
(b)	States for example when $x=-8, y=-2$, $\frac{x+y}{2}=-5$, $\sqrt{xy}=4$ so	B1
	$\frac{x+y}{2}\leqslant\sqrt{xy}$	**(1)**
		(4 marks)

Question Number	Scheme	Marks
3. (i)	$(x-4)^2 \geqslant 2x-9 \Rightarrow x^2-10x+25 \ldots 0$	M1
	$\Rightarrow (x-5)^2 \ldots 0$	A1
	Explains that "square numbers are greater than or equal to zero" hence (as $x \in \mathbb{R}$), $\Rightarrow (x-4)^2 \geqslant 2x-9$ *	A1*
		(3)
(ii)	Shows that it is not true for a value of n Eg. When $n=3$, $2^n+1=8+1=9$ × Not prime	B1
		(1) **(4 marks)**

Question Number	Scheme	Marks
4 (a)	-35	B1
		(1)
(b)	Attempts $f\left(\pm\frac{2}{3}\right)=0 \rightarrow \left(\frac{2}{3}-3\right)\left(3\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)+a\right)-35=0$	M1
	$-\frac{7}{3}(2+a)=35 \Rightarrow a=-17$ *	A1*
		(2)
(c)	$f(x)=(x-3)(3x^2+x-17)-35=3x^3-8x^2-20x+16$	B1
	$=(3x-2)(x^2-2x-8)$	M1 A1
	$=(3x-2)(x+2)(x-4)$	M1 A1
		(5) **(8 marks)**