10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

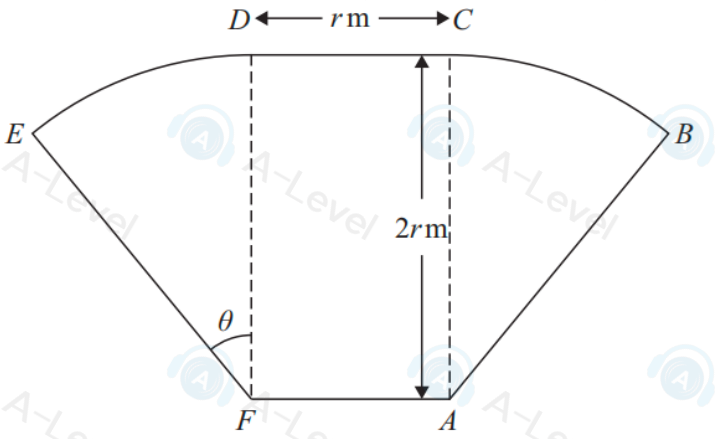

Figure 2

Figure 2 shows the plan view of the design for a stage at a trade fair.

The shape of the stage $ABCDEFA$, consists of a rectangle $ACDF$ joined to two congruent sectors of circles. ABC is a sector of a circle centre A and FDE is a sector of a circle centre F.

Given that $AC = 2r$ metres, $CD = r$ metres, angle $DFE = \theta$ radians and the area of the stage is $30\,\text{m}^2$,

(a) show that the perimeter, P metres, of the stage, is given by

$$P = 4r + \frac{30}{r}$$

(4)

(b) Use calculus to find the minimum value for P, giving your answer in the form $a\sqrt{b}$, where a and b are integers to be found.

(4)

(c) Justify that the value of P found in part (b) is the minimum.

(2)

(Total for Question 10 is 10 marks)

7.

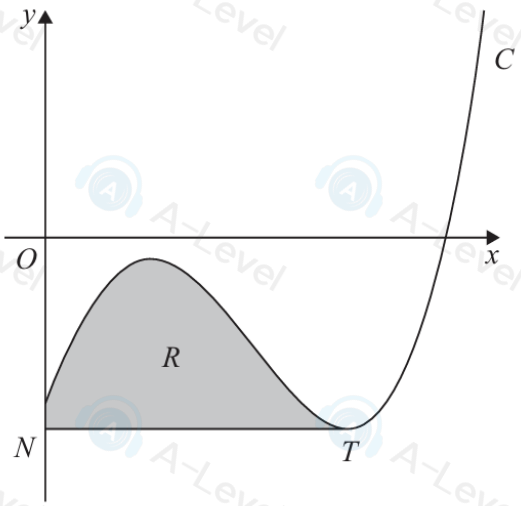

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\dfrac{dy}{dx}$ in simplest form.

(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.

(6)

blank

3. (i) Use algebra to prove that for all real values of x

$$(x-4)^2 \geqslant 2x - 9$$

(3)

(ii) Show that the following statement is untrue.

$2^n + 1$ is a prime number for all values of n, $n \in \mathbb{N}$

(1)

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3 + 2x)^6$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2 - \frac{1}{6x}\right)(3 + 2x)^6$$

(3)