

8. (i) A student writes the following statement:

“When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10”

Prove by counter example that this statement is **not** true.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

4.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + ax^2 - 29x + b$$

where a and b are constants.

Given that $(2x + 1)$ is a factor of $f(x)$,

(a) show that

$$a + 4b = -56$$

(2)

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is -25

(b) find a second simplified equation linking a and b .

(2)

(c) Hence, using algebra and showing your working,

(i) find the value of a and the value of b ,

(ii) fully factorise $f(x)$.

(5)

2.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 - 8x^2 + 5x + a$$

where a is a constant.

Given that $(2x - 3)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $a = -3$

(2)

(b) Hence show that the equation $f(x) = 0$ has only one real root.

(4)

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9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P .

- (b) Show that the x coordinate of P is $\frac{\log_{10} 2}{\log_{10} 24}$

(5)

blank

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b \log_2 3$, where a and b are constants to be found.

(4)

- (ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2 \log_5(y + 1) - 1$$

(5)

10. Given $a = \log_2 3$

- (i) write, in simplest form, in terms of a ,

(a) $\log_2 9$

(b) $\log_2 \left(\frac{\sqrt{3}}{16} \right)$

(3)

- (ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a .

(4)

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4. (a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

- (b) find the possible values of p .

(4)

blank

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7. (a) Given that

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

- (b) Show that -4 is a root of this cubic equation.

(2)

- (c) Hence, using algebra and showing each step of your working, solve

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

blank

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- 4: The function $f(x)$ is defined by

$$f(x) = ax^3 + bx^2 + 5x - 3$$

where a and b are constants.

Given that $(x + 3)$ is a factor of $f(x)$,

- (a) show that

$$b = 3a + 2$$

(2)

Given further that when $f(x)$ is divided by $(2x - 1)$ the remainder is $\frac{7}{4}$

- (b) find the value of a and the value of b .

(4)

- (c) Using algebra, find the quotient and the remainder when $f(x)$ is divided by $(x - 2)$

(3)

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4. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\left(2 - \frac{1}{4}x\right)^6$$

(4)

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = a + bx^2$$

where a and b are constants to be found.

(3)

5.

$$f(x) = 3x^3 + ax^2 - 10x + b$$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $f(x)$,

(a) show that $16a + 9b = 56$

(2)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is b ,

(b) find the value of a and the value of b .

(4)

(c) Hence, using algebra, fully factorise $f(x)$.

(3)

3.

$$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6 \tan^3 \theta + 17 \tan^2 \theta + 4 \tan \theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

blank

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3. (a) Find the first 4 terms, in ascending powers of x , in the binomial expansion of

$$\left(1 + \frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form.

(3)

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2 + 8}{x^5}\right)\left(1 + \frac{x}{4}\right)^{12}$$

(3)

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