9.

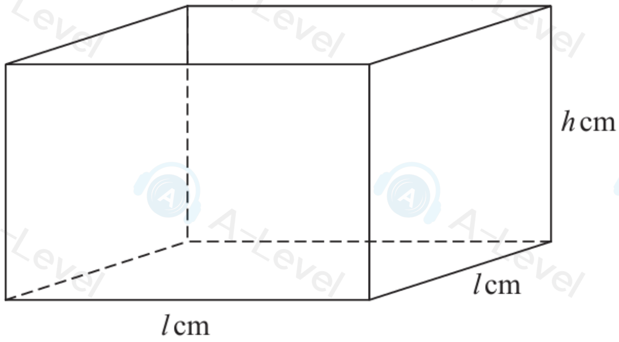

Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is $250\,000\text{ cm}^3$

(a) show that the external surface area, $S\text{ cm}^2$, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

9.

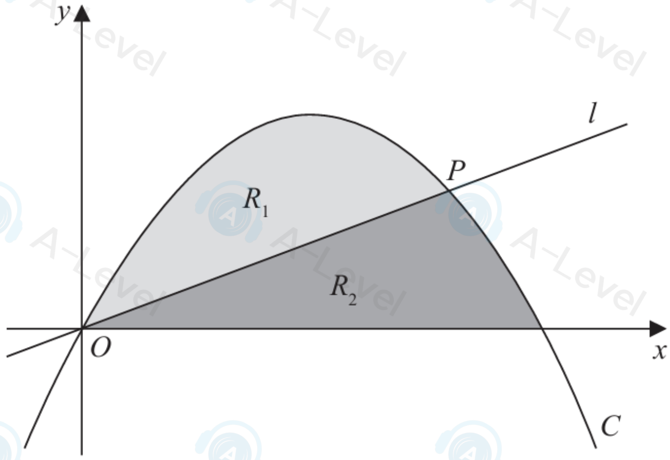

Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

9.

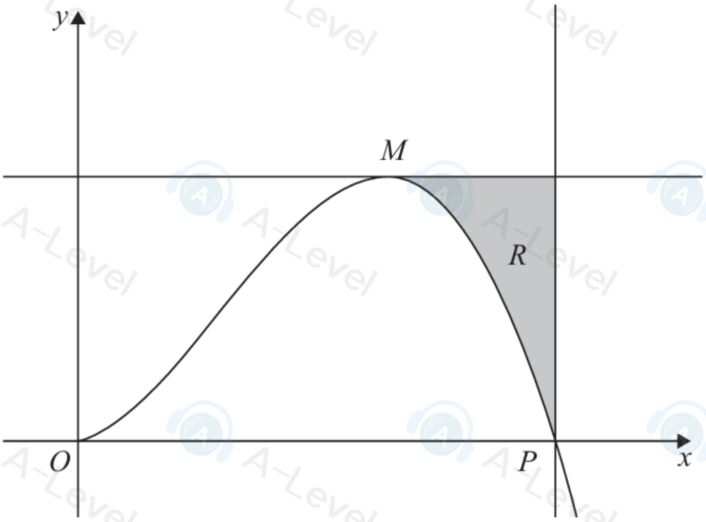

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

2. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

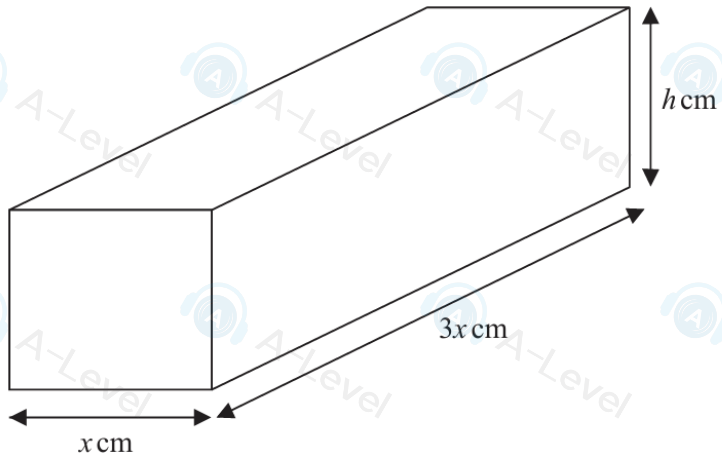

Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is 972 cm^3

(a) Show that the surface area of the brick, $S\text{ cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x}$$

(3)

(b) Find $\frac{dS}{dx}$

(1)

(c) Hence find the value of x for which S is stationary.

(2)

(d) Find $\frac{d^2S}{dx^2}$ and hence show that the value of x found in part (c) gives the minimum value of S.

(2)

(e) Hence find the minimum surface area of the brick.

(1)

8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

3.

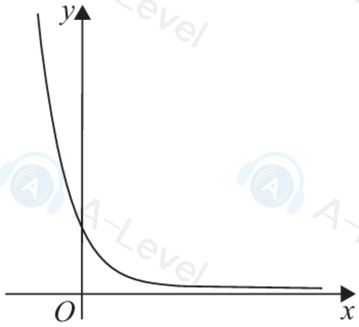

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = 3 \times 2^{-x}$

The point $P(k, 300\,000)$ lies on the curve.

(a) Use logarithms to find the value of k to 2 decimal places.

(2)

x	–0.5	1	2.5	4.0	5.5	7
y	4.243	1.5	0.530	0.188	0.066	0.023

The table shows corresponding values of x and y for $y = 3 \times 2^{-x}$

The values of y are given to 3 decimal places where appropriate.

(b) (i) Use the trapezium rule, with all the values of y from the table, to find an approximate value, to 2 decimal places, for

$$\int_{-0.5}^{7} 3 \times 2^{-x}\, dx$$

(3)

(ii) Use your answer to part (b)(i) to estimate

$$\int_{-0.5}^{7} 2^{-x}\, dx + \int_{-7}^{0.5} 2^{x}\, dx$$

(2)

blank

4.

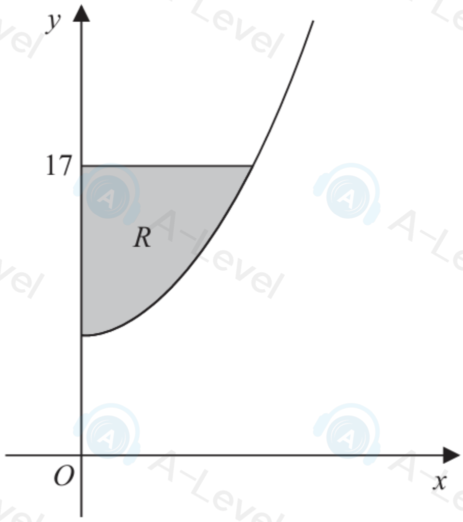

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = 2x^2 + 7 \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis and the line with equation $y = 17$

Find the exact area of R.

(6)

blank

8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

(c) Calculate the value of N according to the model.

(3)

4:

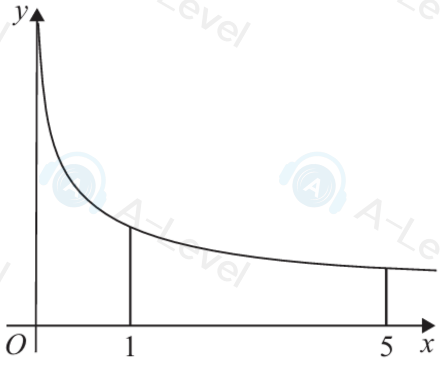

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \log_3(x+1) - \log_3 x$$

The point $P(a, 4)$ lies on the curve.

(a) Find the exact value of the constant a.

(Solutions relying on calculator technology are not acceptable.)

(4)

(b) Use the trapezium rule with 4 strips of equal width to estimate the value of

$$\int_1^5 \left(\log_3(x+1) - \log_3 x\right) \mathrm{d}x$$

giving the answer in the form $\log_3 k$, where k is a constant to be found.

(4)

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for

$$\int_1^5 \left(\log_3(x+1) - \log_3 x\right) \mathrm{d}x$$

(1)

7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a.

(2)

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)