10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}\qquad\theta\neq\frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x+\frac{2}{\tan x}\right)=4\sin x-5$$

giving your answers to 3 significant figures.

(4)

9: The circle C

- has a centre which lies on the x-axis
- touches the y-axis
- passes through the point (5, 6)

(a) On Diagram 1, sketch a graph of C.

(1)

(b) Find an equation for C.

(4)

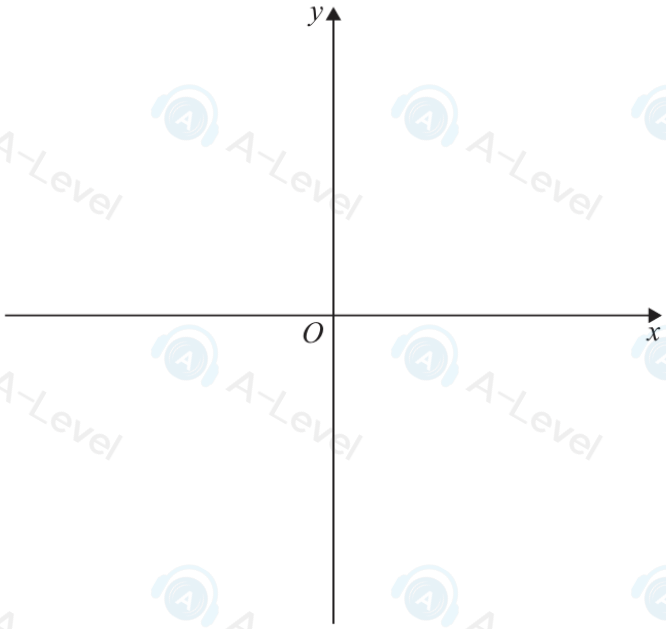

Diagram 1

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

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3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Solve, for $0 < \theta \leqslant 360°$ the equation

$$2\tan\theta + 3\sin\theta = 0$$

giving your answers, as appropriate, to one decimal place.

(5)

(b) Hence, or otherwise, find the smallest positive solution of

$$2\tan(2x + 40°) + 3\sin(2x + 40°) = 0$$

giving your answer to one decimal place.

(2)

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5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 360°$, the equation

$$4\tan\theta + 5\sin\theta = 0$$

giving any non-exact answers to one decimal place.

(5)

(ii) Solve, for $0 < x < \pi$, the equation

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\cos x}$$

giving the answers, in radians, to 3 significant figures.

(4)

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6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Given that θ is measured in degrees and

- $\cos\theta = \frac{1}{\sqrt{5}}$
- $180° < \theta < 360°$

use trigonometric identities to find the exact value of

(a) $\sin\theta$

(b) $\tan\theta$

giving the answers as fully simplified surds where appropriate.

(4)

(ii)

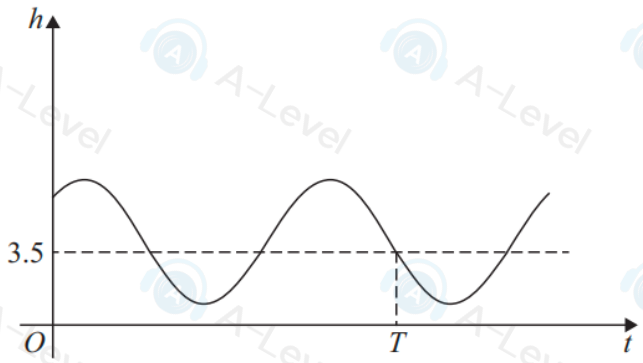

Figure 2

The height of sea water, h metres, on a harbour wall, t hours after midnight on a particular day is given by

$$h = 4 + 3\cos(30t - 40)° \qquad 0 \leqslant t < 24$$

A sketch of h against t is shown in Figure 2.

(a) Find the minimum height of sea water on the harbour wall.

(1)

(b) Find the exact time of day when this minimum height **first** occurs.

(3)

When $t = T$, as shown in Figure 2, a boat enters the harbour when the height of sea water on the harbour wall is 3.5 m.

(c) Use Figure 2 and the given equation to find the value of T to 2 decimal places.

(4)

2. A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

Find

(a) (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The point $P(5, 4)$ lies on C.

(b) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta + \frac{2}{\tan\theta}\right) \equiv \sin\theta + \frac{2}{\sin\theta} \qquad \theta \neq \frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x + \frac{2}{\tan x}\right) = 4\sin x - 5$$

giving your answers to 3 significant figures.

(4)

6.

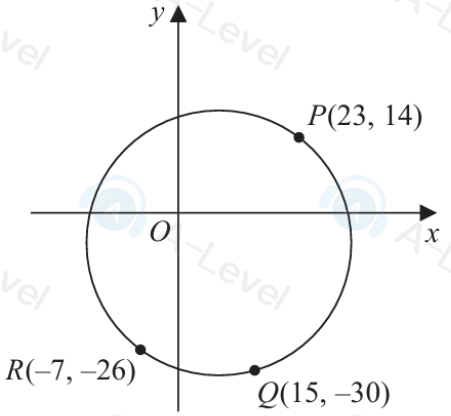

Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

3. A circle has equation

$$x^2 + y^2 + 8x - 14y - 79 = 0$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is nearest the origin O,

(b) find the exact length of OP

(2)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

(ii)

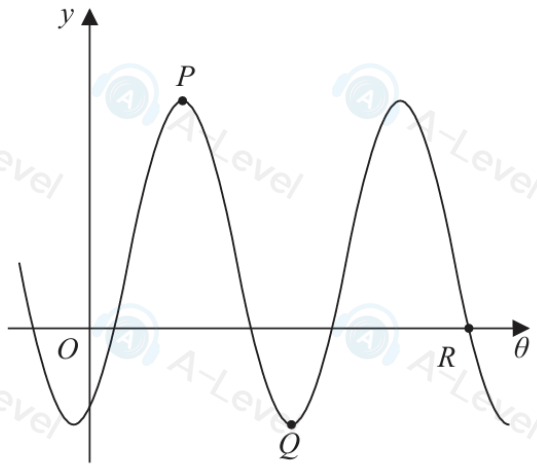

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

blank

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)