7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$ **(1)**

(b) Hence find the value of a. **(2)**

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(Solutions based entirely on graphical or numerical methods are not acceptable.) **(4)**

DO NOT WRITE IN THIS AREA

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The binomial expansion, in ascending powers of x, of $(1 + kx)^n$ is

$$1 - 24x + 270x^2 + px^3 + \ldots$$

where k and p are constants and n is an integer greater than 1

(a) Use this information to set up and solve two simultaneous equations to find the value of k and the value of n. **(6)**

(b) Hence find the value of p. **(2)**

DO NOT WRITE IN THIS AREA

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3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give each term in simplest form. **(4)**

$$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k. **(3)**

DO NOT WRITE IN THIS AREA

5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x + 1) = 3 + \log_2(x + 6)$$

(5)

DO NO

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2. A sequence is defined by

$$u_1 = 6$$
$$u_{n+1} = ku_n + 3$$

where k is a positive constant.

(a) Find, in terms of k, an expression for u_3 **(2)**

Given that $\sum_{n=1}^{3} u_n = 117$

(b) find the value of k. **(3)**

6. A circle C has equation $x^2 + y^2 - 6x - 14y + k = 0$ where k is constant.

(a) Find the coordinates of the centre of C. **(2)**

(b) Find the radius of C when $k = -32$ **(2)**

(c) Find the range of values of k for which C lies completely within the first quadrant. **(4)**

(Total for Question 6 is 8 marks)

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3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$\left(1 + \frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form. **(3)**

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2 + 8}{x^5}\right)\left(1 + \frac{x}{4}\right)^{12}$$

(3)

9.

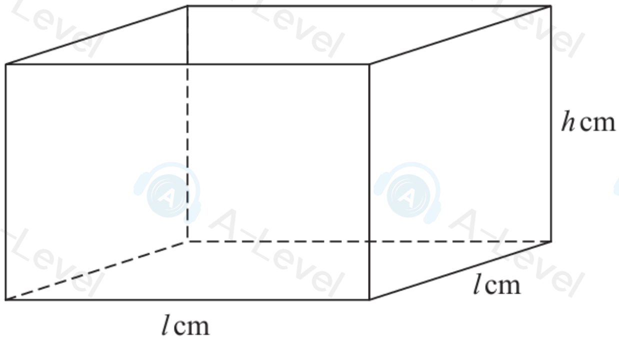

Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is 250 000 cm^3

(a) show that the external surface area, S cm^2, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$

(2)

(ii) $\dfrac{\log_a a^2 b}{\log_a b^3}$

(2)

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$

(3)

3. (i) Show that the following statement is **false**:

"$(n+1)^3 - n^3$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve

$$3^a = 70$$

giving the answer to 3 decimal places.

(2)

(ii) Find the exact value of b such that

$$4 + 3\log_3 b = \log_3 5b$$

(4)